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Sparse Representation through Multi-Resolution Transform for Image Coding

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Abstract

Having a compact basis is useful both for compression and for designing efficient numerical algorithms. In this paper, a new **image** coding scheme using a multi-resolution transform known as Bandelet Transform that provides an optimally compact basis for images by exploring their directional characteristics is proposed. As this process results in a sparse representation, Zero Vector Pruning is applied in-order to extract the non-zero coefficients. Further the geometric interpixel redundancies present in the transformed coefficients are removed. The psycho-visual redundancies are removed using simple Vector Quantization (VQ) process. Finally, Huffman encoder is used to encode the significant coefficients. The proposed compression method beats the standard wavelet based algorithms in terms of mean-square-error (MSE) and visual quality, especially in the low-rate compression regime. A gain in the bit-rate of about 0.81 bpp over the wavelet based algorithms is achieved yielding similar quality factor.

Keywords: Bandelet Transform, Multi- Resolution, Psycho-Visual Redundancy, Vector Quantization, Zero Vector Pruning.

1. Introduction

Sparse representation and multi-resolution properties [1] have been utilized in the computing field to speed up various numerical operations [2, 3]. Sparse representation of the images results in faster computation of their linear combinations since only the non-zero coefficients are considered. Multi-resolution makes it easy to perform the warping operation at multiple resolutions, as well as in a coarse-to-fine fashion. The wavelet transform [4, 5] has emerged as the preeminent tool for image modelling. The success of wavelets is due to the fact that they provide a sparse representation for smooth signals interrupted by isolated discontinuities [6] (this is a perfect model for ‘image slices’- 1D cross sections of a 2D image).The success of wavelets does not extend to 2D images. Although wavelet-based images processing algorithms define the state-of-the-art, they have significant shortcomings in their treatment of edge structure. No matter how smooth a contour is, large wavelet coefficients cluster around the contours, their number increasing at fine scales. The wavelet transform is not sparse for images that are made up of smooth regions separated by smooth contours [7]. It simply takes too many wavelet basis functions accurately to build up an edge. In short, the wavelet transform makes it easy to model grayscale regularity, but not geometric regularity.

Several recently proposed directional approaches use the lifting scheme in image compression algorithms. This scheme has been exploited by Gerek and Cetin [8] where transform directions are adapted pixel-wise throughout images. A similar adaptation is used by Chang and Girod [9] but with fixed number of directions. However, even though these methods are computationally efficient and provide good compression results, they show a weaker performance when combined with zero-tree based compression algorithms. To enhance wavelets representations, Ding et al. [10] have proposed to approximate the wavelet coefficients using adaptive vector quantization [11] techniques. Following the work of Sweldens [12] on adaptive lifting schemes, new lifting algorithms have also been proposed to predict wavelet coefficients from their neighbors. These works are mostly algorithmic and do not provide mathematical bounds. They use the fact that wavelet coefficients inherit some regularity from the image geometric regularity. Filter Bank Techniques uses windowing of the sub-band coefficients which may lead to blocking effects. To overcome this problem, Do and Vetterli [13] proposed the Pyramidal Directional Filter Bank (PDFB). This approach overcomes the block based approach of the curvelet by using a directional filter bank [14].

The proposed work concentrates on modelling the geometric regularity in images using a simple new decomposition, the bandelet representation. This multiscale geometry model captures the joint behavior of the bandelet orientations along the direction of a geometric flow. This geometric flow indicates the direction in which the image grey levels have regular variations. The image decomposition in a bandelet basis is implemented with a fast sub-band filtering algorithm.

In Section II, the construction of bandelet bases allowing for geometrical flow and multi-directionality is presented. The bandeletization process is discussed in Section III. Section IV analyzes the asymptotic rate-distortion performance of a coder based on the geometric optimization using bandelets and Section V reports experimental results of coding and the performance comparison with the existing methods. Conclusion is given in Section VI..

2. Ban delete Basics

A bandelet is a piecewise constant function on a square domain Ω_i that is discontinuous along a line passing through Ω_i . It combines multi-resolution theory with geometric partitioning of the image domain [15], which makes use of redundancies in the geometric flow, corresponding to local directions of the image grey levels considered as a planar one dimensional field. The geometry of the image is summarized with local clustering of similar geometric vectors, the homogeneous areas being taken from a quad tree structure.

Bandelet representation of an image X consists of a dyadic partition of the domain of X along with a bandelet function in each dyadic square. In each square the geometry is defined by finding not an edge location but an orientation along which the image has regular variations. This orientation is defined by a vector field called geometric flow, which is nearly parallel to the edge. To take advantage of the image regularity along the flow, a larger image band parallel to the flow is warped into each dyadic square. This warped image is decomposed over an anisotropic separable orthogonal basis. Le Pennec and Mallat [16] proved that decomposing a geometrically-regular image over a best bandelet frame yields a bandelet approximation that satisfies $\|X - X_C\|^2_{\Omega^2} = O(C^{-\beta})$ where C is the total number of bandelet coefficients that specifies the geometric flow. A Lagrangian minimization computes a bandelet basis *B* whose segmentation and geometric flows are adapted to the image X. For image compression and noise removal applications, the geometric flow is optimized with fast algorithms, so that the resulting bandelet basis produces a minimum distortion with $O(n^2(\log_2 n^2))$ operations for an image of n^2 pixels, because the geometry is structured by aggregating nearly independent building blocks. This optimization requires establishing the link between the image geometry and the distortion-rate of the image coder.

3. The Bandeletization Process

The input image is decomposed using the Warped Haar Transform based on an orthogonal basis formed by the translation and dilation of three mother wavelets for the horizontal, vertical and diagonal directions. Once the transform is applied, the quad-tree is computed by dividing the image into dyadic squares. For each square in the quad-tree the optimal geometrical direction is computed by the minimization of a Lagrangian. The Lagrangian approach proposed by Ramachandran and Vetterli [17] finds the best basis that minimizes $dR + \lambda R$, where λ is a Lagrange multiplier, d is the Distortion and R is the number of bits. If dR is convex, which is usually the case, by letting λ vary dR shall be minimized. If dR is not convex, then this strategy leads to a dR that is at most a factor 2 larger than the minimum. Even in squares with no geometric features (on which the function is constant), the algorithm chooses some arbitrary orientation. This is because in these squares the function does not have zero mean, so a bandeletization (with any direction) is better than leaving the data untransformed [18]. This situation does not appear in the wavelet-bandelet algorithm, since in flat areas, a wavelet transform has zero mean. Then a projection of the transform coefficients along the optimal direction is performed. Finally a 1D haar transform is carried on the projected coefficients. Particularly, the size and the optimal geometrical direction of each square will be used as criteria to study the similarity. The inverse discrete bandelet transform computes the image values on the original integer sampling grid (m,n) from the sample values $V_i[k_1,k_2]$ along the flow lines in each Ω_i where $(k_1,k_2) \in z^2$.

4. The Proposed Coding Scheme

The input image is decomposed into the bandelet basis. This process introduces psycho-visual and inter-pixel redundancies by integrating the geometric regularity in the image representation. Hence the bandeletized coefficients are subject to zero vector pruning (ZVP) process to reduce the inter-pixel redundancy. ZVP identifies all non-zero vectors along with their row indices thereby eliminating redundant zero vectors. Then the correlation coefficient is used to identify the sequential pattern present in the input vectors to the quantization stage. The correlation coefficient r is given in Eq.1.

$$r = \frac{\sum_m \sum_n (A_{mn} - \overline{A})(B_{mn} - \overline{B})}{\sqrt{\left(\sum_m \sum_n (A_{mn} - \overline{A})^2 \sum_m \sum_n (B_{mn} - \overline{B})^2\right)}} \quad \ldots(1)$$

where A, B are the row/column vectors in the transformed input matrix; $\overline{A}, \overline{B}$ are the respective mean values of the row/column vectors A, B. To find out the sequential pattern, the concept of point estimation is used.

Point estimation refers to the process of estimating a population parameter (e.g. correlation), by actually calculating the parameter value for a population sample [19]. Initially, A is assigned with the first row/column vector of the input to this stage. B is assigned with the second input vector. The correlation between A and B is computed using Eqn. (1). A threshold value (τ) is used to identify the correlation between A & B. If $0.8 < \tau < 1$, then A & B are said to be highly correlated. Then, the third vector is assigned to B and the process of computing the correlation is repeated. This process of comparison is repeated for few input vectors and then the sequential patterns present in the input matrix are determined. These patterns are indexed. Now the input vectors are quantized by assigning the indices of the corresponding sequential pattern to which they are the closest. The quantized coefficients are coded with Huffman encoder. The compressed image is decompressed using Huffman decoder, vector reassignment procedure followed by the reverse Bandelet transform to reconstruct the image. The flow of the proposed work is depicted in Figure 1.

5. Experimental Results

To evaluate the performance of this bandelet based compression algorithm a comparison is made with the same coder applied to a wavelet and wavelet packet-based compression. Figure 2a shows 512 × 512 Barbara Original Image. Figure 2b shows the respective reconstructed image that is obtained using the proposed Image Coder. To illustrate the effectiveness of the proposed Coder, the enlarged face portion of the Barbara Original Image is shown in Figure 2c, and the respective reconstructed image portions using the proposed Coder, the existing Wavelet based

Multistage Vector Quantizer (W-MSVQ) [20] and Wavelet Packet based Lindo-Buzo-Gray Coder (WP-LBG) are shown in Figures 2d, 2e and 2f. It is observed from the figure (Figure 2e.) that W-MSVQ suffers from more pronounced blocking artifacts. Though the effect of blocking artifacts is reduced using WP-LBG (Figure 2f.) it is observed that this method suffers from smoothening effect and hence ignoring the detail information. The proposed Image Coder (Figure 2d.) because of its geometry preserving nature preserves details and reduces blocking artifacts seen in the reconstructed image thereby improving the subjective psycho-visual quality remarkably. Barbara image is purposely chosen as the test image since it contains more detail information which helps in the measure of the subjective evaluation of the quality of the reconstructed images using the proposed and other existing methods.

Two issues are to be addressed before the implementation phase. They are the choice of the Threshold (T) for Lagrangian computation and a Scale Factor (SF) selection for Bandeletization.

A. *Threshold (T)*

The impact of the Threshold (T) for Lagrangian computation is depicted in a graph shown in Figure 3. The observations are recorded by varying the threshold (T) from 0 to 2 in steps of 0.2. It is observed that PSNR increases as T is varied from 0 to 0.4 and then decreases with further increase in T. Therefore the gain in PSNR is maximum with T=0.4.

B. *Scale Factor (SF)*

Selecting an optimal value for the SF is influenced by parameters like Compression Ratio and the computation Time (Figure 4). Figure 4.a. shows the impact of SF on the gain in quality (PSNR) and Compression Ratio. The quality of the reconstructed image is not affected apparently. But the compression ratio varies from 1: 11.54 to 1: 3.79 as the scale factor is varied logarithmically as shown for the Barbara image. It shall be noted from Figure 4.b. that the Computation Time increases as the SF increases logarithmically. The performance of the proposed work is analyzed with T = 0.4 and SF = $\log_2 1$. The graph shown in Figure 4.c depicts the performance of the proposed work by varying the size of the original image. The results are tabulated for various images of size 512 x 512 in Table 1. Compression Ratio is the ratio of the input image size to the compressed image size. Space saving gives the amount of memory space saved due to compression. It is given in Eq.2.

$$Space\,Saving = \left(1 - \left(1/C_R\right)\right) \times 100\,; \qquad \ldots (2)$$

where, C_R is the Compression Ratio.

It is perceived from this table that on an average the proposed work gives a Compression Ratio of 1:11 leading to 1.4 bits per pixel representation for the compressed file

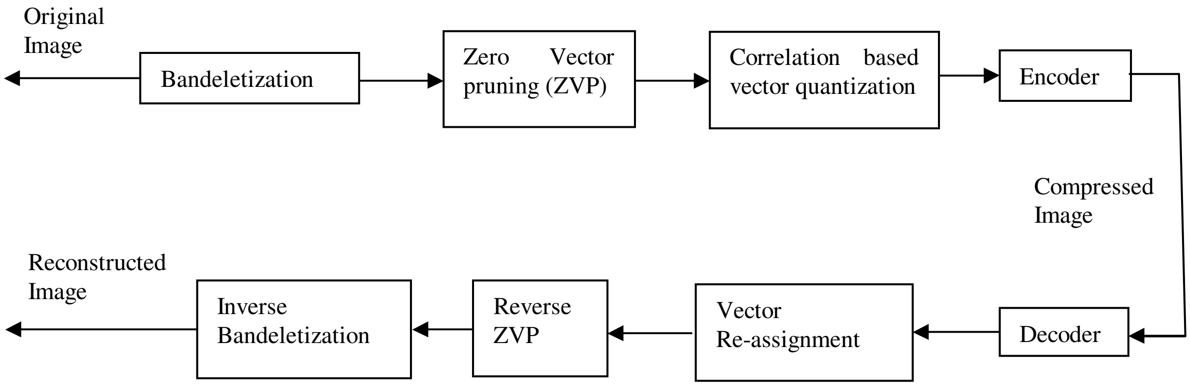

Figure 1: The Proposed Coding Scheme

Figure 2a. Original Image

Figure 2b. Reconstructed Image using the proposed method

Figure 2c. Enlarged Face portion of the Original Image

Figure 2d. Reconstructed Face portion using the proposed model

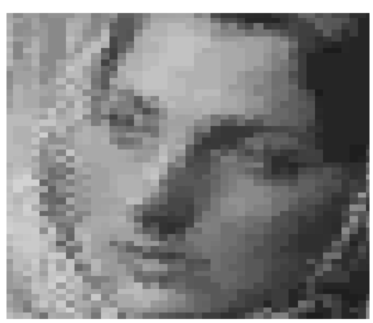

Figure 2e. Reconstructed Face portion using W-MSVQ

Figure 2f. Reconstructed Face portion using WP-LBG

with 90% space saving with an average PSNR value to about 28 dB. The performance comparison of the proposed work with the existing methods for the Barbara image is illustrated in Table 2. The proposed work outperforms the existing methods, giving out a high Compression Ratio of 1: 9.96 resulting in a gain in the Bit rate of 0.81 bpp with 24.42 db quality factor for the Barbara image.

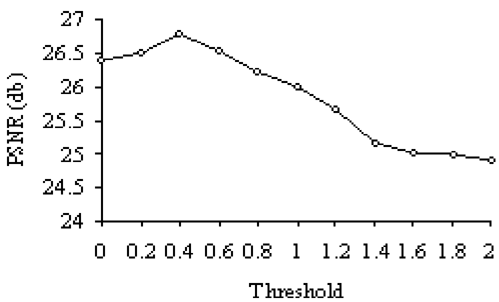

Figure 3. Threshold vs Image Quality

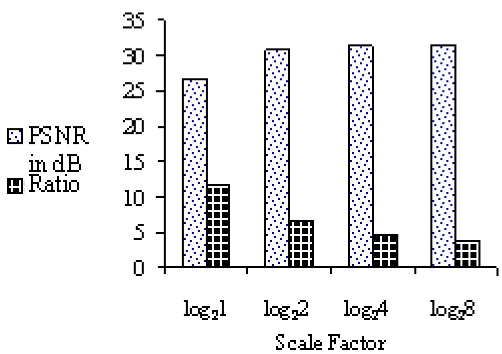

Figure 4a. Scale Factor Vs PSNR & Ratio

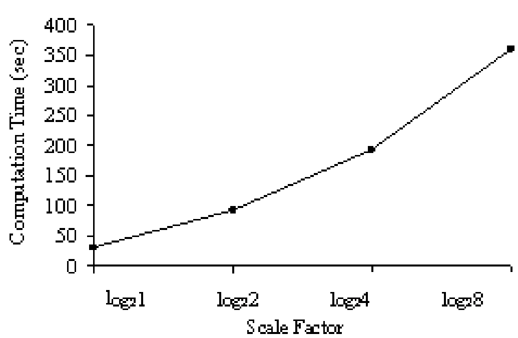

Figure 4b. Scale Factor vs Computation Time

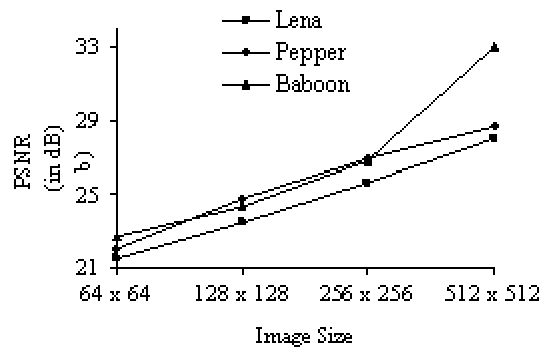

Figure 5a. Image Size vs PSNR

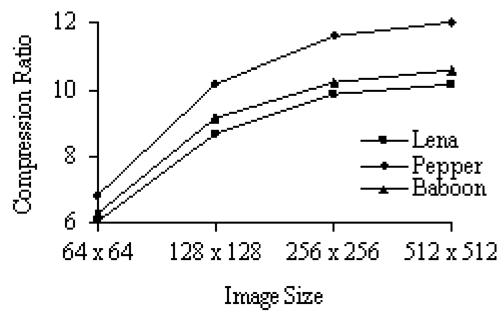

Figure 5b.Image Size Vs Ratio

6. Conclusion

A simple way of computing various two dimensional image distortions in the bandelet domain is presented in this paper. Bandelets retain critical sampling and the simplicity of the filter design from the standard wavelet Transform. As the process of bandeletization allows for sparser representations of the directional anisotropic features, bandelets are applied in the approximation and compression methods based on Lagrangian optimization. To remove the sparsity, redundancy removal techniques using correlation coefficient and point estimation concepts are used. Finally, the proposed bandelet based model obtained as a combination of Bandelets, Quantization and Coding outperforms the state-of-the-art methods in terms of both the numerical criterion and the subjective visual quality. In future it is planned to apply this approach to other types of image and video operations, such as image warping (perhaps using more complicated mappings), and blending of image sequences.

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