Direct Heuristic Algorithm for Linear Programming

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Abstract

Many applications in business and economics involve a process called optimization, in which we will be required to find the minimum cost, the maximum profit, or the minimum use of resources, where a decision maker may want to utilize limited available resources in the best possible manner. The limited resources may include material, money, manpower, space and time. Linear Programming provides various methods of solving such problems. The formulation of linear programming problem as a mathematical model is one type of optimization problem called linear programming. There are various methods for solving the linear programming problems. Some of them are approximation algorithm , branch and bound methods, cutting plane method etc. Other than Gomory's cutting plane method, Branch and bound method LPP along with the DHALP (Direct heuristic algorithm for linear programming) algorithm which is more efficient than these existing methods will be used for solving linear programming problems. An optimality test will also be included in this. Numerical experiments will depict the utility/scope of such a procedure.

Keywords: *Linear programming, maximization, minimization, direct heuristic algorithm, interval valued linear fractional programming problem (IVLFP), Index array, least square inverse, norm, optimal solution, linear programming problem.*

1. Introduction

Mathematical programming is used to find the best or optimal solution to a problem that requires a decision or set of decisions about how best to use a set of limited resources to achieve a state goal of objectives.

Steps involved in mathematical programming are

- Conversion of stated problem into a mathematical model that abstracts all the essential elements of the problem.
- Exploration of different solutions of the problem.
- Finding out the most suitable or optimum solution.

In passing [1] a mathematical formulation of bi-level linear programming problem to deal with an interval number programming approach. Bi-level linear programming problem was usually viewed as a problem with two decision makers at two different hierarchical levels. The upper-level decision maker, the leader, selects his or her decision vector first and the lower decision maker, the follower, selects his or her afterward based on the decisions of the upper level. In mathematical programming problem, the coefficients in the objective function and the constraint functions were always determined as crisp values. In practice, however, there were many decision situations where the objective functions and/or the constraints were uncertain to some degree. Over the last two decades, interval programming based on the interval analysis had been developed as a useful and simple method to deal with this type of uncertainty. The interval numbers were in both of the objective function and the constraints. An Illustrative numerical example was provided to clarify the proposed approach.

1.1 Solution Algorithm

In passing [2] a solution algorithm that had been proposed to solve fuzzy integer linear fractional programs (FILFPP). Some fuzzy concepts had been given to convert problem (FILFPP) to a no fuzzy version and the Charnes & Cooper transformations had been used to complete the solution process. Summarizing, many aspects and general questions remained to be studied and explored in the area of fuzzy integer linear fractional programming.

Despite the limitations, they believed that this was an attempt to establish underlying results which hopefully helped others to answer of the questions. There were however several open points for future research in the area of (FILFPP).

1.2 Hybrid Heuristic Algorithm

In passing [3] a hybrid heuristic algorithm, which used procedures for search of feasible integer directions with one or two nonzero components and linear optimization. The algorithm was iterative and it combined constructive and locally improved strategies for finding a new current solution. A subsequence of sub problems was solved, aimed at seeking a feasible solution of the general Mixed Integer Problem (MIP), after which the feasible solution found was improved with respect to the problem objective function. The algorithm was characterized by polynomial-time computing complexity.

In passing [4] a survey of methods and approaches solving linear integer problems, developed during the last 50 years. A large variety of different real life problems in practice were formulated as integer optimization problems. Their number and their size increased continuously. Regardless of the fact, that the productivity of exact algorithms designed to solve integer problems had been considerably improved during the last years, very often they were not be applied to solve practical problems of middle and large size because of their excessive runtimes and memory requirements. The published theoretical and also algorithmic investigations were devoted to combinatorial or binary problems. As a result, the most wide spread heuristic procedures for obtaining suitable initial solutions, evaluations of candidate-solutions, cutting planes, specialized search strategies, etc., integrated in the commercial programming products, were effective for problems with 0-1 variables or for problems having a special structure. The solution of the integer problem in the general case remained considerably harder. The hybrid methods were promising tools, since they combined the best features of different methods (exact techniques or Meta heuristics) in a complementary mode. Those problems belonged to the class of NP-hard optimization problems. To find out exact optimal solutions for this class of problems it required the use of considerable computational resources. The development of efficient hybrid methods, combining in a suitable way the best features of different approaches (exact or approximate) was the actual direction, in which many researchers devoted their efforts to solve successfully various hard practical problems. Many large size real problems were not solved by exact algorithms due to their exponential computational complexity. In such case the only way was to use the approximate polynomial time algorithms.

1.3 FUZZY DECISIVE SET METHOD

In passing [5] a fuzzy multi-objective linear programming problem in which both the resources and the technological coefficients were fuzzy with linear membership function was studied. Further a FMLOP problem was converted into an equivalent crisp non-linear programming problem using the concept of max-min principle. The resultant non-linear programming problem was solved by fuzzy decisive set method. The discussed method was illustrated through an example. The method could be extended to solve problems like FMLOP with triangular or trapezoidal membership function and linear fuzzy fractional programming problems.

1.4 Exponential Barrier Method

Explicitly as in [6] concerned with the study of the exponential barrier method for linear programming problems with the essential property that each exponential barrier method is concave when viewed as a function of the multiplier.. It presented some background of the method and its variants for the problem. Under certain assumption on the parameters of the exponential barrier function, they gave a rule for choosing the parameters of the barrier function. Theorems and algorithms for the methods were also given.

1.5 Interval Valued Programming Approach

In passing [7] an interval valued goal programming approach for solving multi objective fractional programming problems. In the model formulation of the problem, the interval-valued system constraints were converted in to equivalent crisp system. The interval valued fractional objective goals were transformed into linear goals by employing the iterative parametric method which was an extension of Dinkelbach approach. In the solution process, the goal achievement function, termed as 'regret function', was formulated for minimizing the unwanted deviational variables to achieve the goals in their specified ranges and thereby arriving at most satisfactory solution in the decision making environment.

In passing [8] an interval valued linear fractional programming problem (IVLFP). An IVLFP is a linear fractional programming problem with interval coefficients in the objective function. It was proved that it can convert an IVLFP to an optimization problem with interval valued objective function which its bounds were linear fractional functions. Also there was a discussion for the solutions of this kind of optimization problem.

In passing [9] a Satisfaction Function (SF) to compare interval values on the basis of Tseng and Klein's idea. The SF estimated the degree to which arithmetic comparisons between two interval values were satisfied. Then, defined two other functions called Lower and

Upper SF based on the SF. Those functions were applied in order to present a new interpretation of inequality constraints with interval coefficients in an interval linear programming problem. This problem was as an extension of the classical linear programming problem to an inexact environment. On the basis of definitions of the SF, the lower and upper SF and their properties, they reduced the inequality constraints with interval coefficients in their satisfactory crisp equivalent forms and defined a satisfactory solution to the problem. Finally, a numerical example was given and its results were compared with other approaches.

1.6 Parametric Approach

In passing [10] an inverse model for linear fractional programming (LFP) problem, where the coefficients in the objective function were adjusted as little as possible so that the given feasible solution x and objective value z becomes optimal. An inverse version of linear fractional programming problem had been studied. The new approach was useful in the situation where the enterprise wanted to work with certain efficiency or wanted to fulfill the sudden market demand with certain efficiency and available resources. This approach was further extended to the nonlinear fractional programming. Here the parametric approach was used for formulating the inverse LFP problem as a linear programming problem. The method had been illustrated by a numerical example also.

In passing [11] with multi objective bi-level linear programming problems under fuzzy environment. In this method, tentative solutions were obtained and evaluated by using the partial information on preference of the decision-makers at each level. The existing results concerning the qualitative analysis of some basic notions in parametric linear programming problems were reformulated to study the stability of multi objective bi-level linear programming problems. An algorithm for obtaining any subset of the parametric space, which had the same corresponding Pareto optimal solution, was presented. Also, it established the model for the supply-demand interaction in the age of electronic commerce (EC). First of all, the study used the individual objectives of both parties as the foundation of the supply-demand interaction. Subsequently, it divided the interaction, in the age of electronic commerce, into the following two classifications: (i) Market transactions, with the primary focus on the supply demand relationship in the marketplace; and (ii) Information service, with the primary focus on the provider and the user of information service. By applying the bi-level programming technique of interaction process, the study had developed an analytical process to explain how supply-demand interaction achieved a compromise or why the process failed. Finally, a numerical example of information service was provided for the sake of illustration

1.7 Linear Programming Model

In passing [15] Linear Programming based Effective Maintenance and Manpower Planning Strategy a Case Study. Linear Programming (LP) model was formulated based on the outcomes of the analyzed data. The data analyzed included maintenance budget, maintenance cycle, production capacity and waiting time of production facilities in case of failure. Data were analyzed based on manpower cost, machine depreciation cost and the spare part cost, which were assumed to be proportion to the number/magnitude of the breakdowns. The generated LP model was solved using software named "the Quantitative System for Business- QSB (Version 3.0). The results of the model showed that four maintenance crews were needed to effectively carryout maintenance jobs in the industry. The sensitivity analysis showed that the results had a wide range of feasibility.

In passing [16] considered two classes of fuzzy linear programming problems: (1) Fuzzy number linear programming (FNLP), and (2) linear programming with trapezoidal fuzzy variables (FVLP) problems. They used the trapezoidal fuzzy numbers and a linear ranking function to describe a fuzzy concept of the basic feasible solutions for both problems. Then used the optimality conditions for the FNLP and the FVLP problems and developed fuzzy primal simplex algorithms for solving these problems. Finally, the solved illustrative examples using the simplex algorithms were presented.

In passing [17] presented Comments on a mixed integer linear programming formulation of the optimal mean/Value-at-Risk portfolio problem. A mixed integer linear programming formulation of the optimal mean/Value-at-Risk portfolio problem, European Journal of Operational Research in a recent proposal of two linear integer programming models for portfolio optimization using Value-at-Risk as the measure of risk, claimed that the two counterpart models are equivalent.

This note showed that this claim was only partly true. The second model attempted to minimize the probability of the portfolio return falling below a certain threshold instead of minimizing the Value-at-Risk. However, the discontinuity of real-world probability values makes the second model impractical. An alternative model with Value-at-Risk as the objective was thus proposed.

In passing [18] proposed equivalence between the feasible set of a bi-level multi objective linear programming and the set of efficient points of an artificial set, in order to find an optimal solution. The second approach used a Pareto-filter scheme to find an approximated discrete representation of the efficient set. The second approach had the advantage to keep the multi criteria concept of the upper DM, while the first one used an aggregation process to eliminate the multi-criteria concept for the leader. The research benefitted the development of decision support systems for tackling bi-level multi objective linear optimization problems in the real world.

1.8 Generic Integer Linear Programming Formulation

In passing [19] presented Generic Integer Linear Programming Formulation for 3D IC Partitioning. The success of 3D IC required novel EDA techniques. Although many EDA techniques exist, this technique focused on 3D IC partitioning, especially at the architectural level to maximize its benefits. First, logical formulations for 3D IC partitioning problems were derived and then the formulations were transformed into integer linear programs (ILPs). The ILP formulation minimized the usage of vertical interconnects subject to the footprint and power consumption constraints. The flexibility of ILP formulation was demonstrated by extending the generic ILP formulation to support designs with multiple supply voltages. This study proposed an ILP reduction technique to speed up the convergence. Experimental results based on the GSRC benchmark showed that our approach converges efficiently. Moreover, the approach was flexible and was readily extended to the partitioning problems with variant objectives and constraints, and with different abstract levels, for example, from the architectural level down to the physical level. This flexibility had made the ILP formulations superior alternatives to 3D IC partitioning problems.

In passing [20] presented Integer Programming Formulations for Maximum Lifetime Broadcasting Problems in Wireless Sensor Networks. The aim was to show that tools like integer linear programming, often regarded as over-theoretical and unrealistic, were indeed suitable frameworks to include the latest advances in energy consumption and communication models in wireless sensor networks. Three models of increasing realism had been presented. Experimental results suggested that integer linear programming was used not only as an effective modeling tool, but also as an efficient solving method for problems of realistic size. A surprising result also indicated that the easiest models to solve (in terms of computation times) were the most realistic ones, suggesting that they should be preferred in general. A speed-up technique, based on the characteristics of the problem, had been discussed and experimentally shown to be effective on many of the problems considered. A practical drawback introduced by the speed up technique had been finally identified and a method to overcome it had been introduced.

1.9 An Algorithmic Approach to Multi Objective Fuzzy Linear Programming Problem

In passing [21] proposed An Algorithmic Approach to Multi Objective Fuzzy Linear Programming Problem. In this, Multi objective Fuzzy Linear Programming Problem under constraints with fuzzy coefficients was considered. A specific ranking method based on distance between fuzzy numbers was used for developing the ranking algorithm. By the Ranking algorithm, MOFLPP using triangular fuzzy numbers was transformed into MOLPP and then solved by Preemptive optimization method. Again it remained to research MOFLPP with general fuzzy numbers and MOFLPP with fuzziness in objective functions.

In passing [22] presented the Solution of Fuzzy Linear Programming Problem. Fuzzy linear programming problem occurred in many fields such as Mathematical modeling, Control theory and Management sciences, etc. In this they presented a new method for solving fuzzy linear programming with fuzzy variables in parametric form. To identify the optimal solution, it had been proposed that the fuzzy linear programming problem was replaced by two auxiliary crisp linear programming problems. Numerical examples were provided to illustrate the method.

In passing [23] proposed a new method for solving a multi-objective linear programming model with fuzzy random variables. In this model, a multi-objective linear programming problem with real variables and fuzzy random coefficients was introduced. Then, a new algorithm was developed to solve the model based on the concepts of mean value of fuzzy random variables, chance-constrained programming and piecewise linear approximation method. Furthermore, a nonlinear programming problem which was obtained by Charnes and Cooper's chance constrained approach had been converted to a mixed integer programming problem by using the piecewise linear approximation method. It also found that the global optimal solution of this nonlinear programming problem was incredibly near to the optimal solution of PLA approach. It considered probability distribution function and the variance effect that had direct effect on optimal solutions and its optimal solution was more confident than other optimal solutions.

Furthermore, an illustrative numerical example was also given to clarify the method.

1.10 Simplex Algorithm

In passing [24] proposed Generalized Simplex Algorithm to Solve Fuzzy Linear Programming Problems with Ranking of Generalized Fuzzy Numbers. In this, the shortcomings of an existing method for comparing the generalized fuzzy numbers were pointed out and a new method was proposed for same. Also using the proposed ranking method, a generalized simplex algorithm was proposed for solving a special type of fuzzy linear programming (FLP) problems. To illustrate the proposed algorithm a numerical example was solved and the advantages of the proposed algorithm were discussed. Since the proposed algorithm was a direct extension of classical algorithm so it was very easy to understand and apply the proposed algorithm to find the fuzzy optimal solution of FLP problems occurring in the real life situations.

In passing [25] proposed Sensitivity Analysis on Linear programming Problems with Trapezoidal Fuzzy Variables. In the real word, there were any problems which have linear programming models and sometimes it was necessary to formulate these models with parameters of uncertainty. Many numbers from these problems were linear programming problems with fuzzy variables. Some authors considered these problems and have developed various methods for solving these problems. Recently, it was considered linear programming problems with trapezoidal fuzzy data and/or variables and stated a fuzzy simplex algorithm to solve those problems. Moreover, they developed the duality results in fuzzy environment and presented a dual simplex algorithm for solving linear programming problems with trapezoidal fuzzy variables. Here, the authors showed that this presented dual simplex algorithm directly used the primal simplex tableau algorithm tenders the capability for sensitivity (or post optimality) analysis using primal simplex tableaus.

In passing [26] fuzzy linear programming problem for trapezoidal number with the help of simplex algorithm and crisp linear system of equation using the linear ranking function. They proposed few methods to find the fuzzy optimal solution of fuzzy programming problem. Here row reduced echelon form of matrices was used to construct a new method for solving FLPP and Fuzzy simplex algorithms for solving fuzzy number linear programming and also used the general linear ranking functions on fuzzy numbers. The methods were very easy to understand and to apply for fully fuzzy linear system occurring in real life situation as compared to the existing methods. A numerical example was solved to illustrate the method and the obtained results were discussed.

1.11 Polynomial Barrier Method

In passing [27] Polynomial Barrier Method for Solving Linear Programming Problems. It had described the barrier functions with barrier terms in polynomial order for solving linear programming problems with the essential property that each member was concave polynomial order even when viewed as a function of the multiplier. Under certain assumption on the parameters of the barrier function, it had given a rule for choosing the parameters of the barrier function. The algorithms for those methods were also given in this. The Algorithm was used to solve the problem. It also noted the important thing of those methods which did not need an interior point assumption.

1.12 DNA Approach

In passing [28] a design and solving LPP method for binary linear programming problem using DNA approach. It was presented with a Bio-process to solve a Binary Linear programming problem using DNA computing approach. It had introduced a solution based methods and the technique in general and the associate frame work that accommodated a number of different feasible solutions to get the optimal. It also helped to make a best possible use of available productive resources such as time, labor, machine etc. In a production process, if bottlenecks occur the Linear Programming highlights the varieties of bottlenecks. Hybridization was performed to extract the required computing output from the combination of Olingonucleotides. Since the applicability and feasibility of DNA computing approach, it was found that the more complex problems of this type of nature could be successfully designed. This problem is also solved manually using LPP Simplex method which had produced the same result.

1.13 Back-Propagation Algorithm

In passing [29] a technique that employed Artificial Neural Networks and expert systems to obtain knowledge for the learner model in the Linear Programming Intelligent Tutoring System(LP-ITS). It also to determined the academic performance level of the learners in order to offer the proper difficulty level of linear programming problems to solve. LP ITS used feed forward Back-propagation algorithm to be trained with a group of learners data to predict their academic performance. Furthermore, LP-ITS used an Expert System to decide the proper difficulty level that was suitable with the predicted academic performance of the learner. Several tests had been carried out to examine adherence to real time data. The accuracy of predicting the performance of the learners was very high and thus stated that the artificial neural network was skilled enough to make suitable predictions.

1.14 Exponential Penalty Methods

In passing [27] Exponential Penalty Methods for Solving Linear Programming Problems. It was concerned with the study of the exponential penalty method for linear programming problems with the essential property that each exponential penalty method was convex when viewed as a function of the multiplier.. It had presented some background of the method and its variants for the problem. Under certain assumption on the parameters of the exponential penalty function, it had given a rule for choosing the parameters of the penalty function. Theorems and algorithms for the methods were also given. At the end, it had given some conclusions and comments on the methods.

1.15 Bound and Decomposition Method

In passing [28] a new method for finding an optimal fuzzy solution for fully fuzzy linear programming problems. A new method namely, bound and decomposition method was proposed to find an optimal fuzzy solution for fully fuzzy linear programming (FFLP) problems. In this method, the given FFLP problem was decomposed into three crisp linear programming (CLP) problems with bounded variables constraints, the three CLP problems were solved separately and by using its optimal solutions, the fuzzy optimal solution to the given FFLP problem was obtained. Fuzzy ranking functions and addition of nonnegative variables were not used and there was no restriction on the elements of coefficient matrix in the proposed method. The bound and decomposition method was illustrated by numerical examples.

1.16 Similarity Measure

In passing [29] on solving intutionistic fuzzy linear programming problem (IFLPP). The concept of intuitionist fuzzy set was viewed as an alternative approach to define a conventional fuzzy set. A New method was introduced to solve IFLPP, where the similarity measures of intutionistic fuzzy sets had been used for determining the composite relative degree of similarity. The score function was used to calculate the ranking function for the objective function. Thus the method was very useful in the real world problems where the product was uncertain.

Thus various algorithms were there for solving the linear programming problems. As there are various drawbacks in these algorithms we go for the proposed one for solving the linear programming problems.

2. Direct Heuristic Algorithm

Heuristics is the study of the methods and rules of discovery and invention. Heuristics are formalized as rules for choosing those branches in a state space that are most likely to lead to an acceptable problem solution.

Heuristics are employed in two cases. They are as follows.

- A problem may not have an exact solution because of its inherent ambiguities.
- A problem may have an exact solution, but the computational cost of finding it may be prohibitive.

A heuristic is only an informed guess of the next step to be taken in solving a problem. Because heuristics use limited information, a heuristic can lead a search algorithm to a suboptimal solution or fail to find any solution at all. More realistic problems (such as those found in expert systems applications, planning, intelligent control, and machine learning) complicate the implementation and analysis of heuristic search by requiring multiple heuristics to deal with different situations in the problem space.

The direct heuristic algorithm makes use of the following steps

1. Input to the direct heuristic algorithm is m, n, $A=[a_{ij}],\quad i=1(1)m;\quad j=1(1)n,\ b=[b_j],$ $j=1(1)m,\quad c=[c_i],\quad i=1(1)n,$ where $i=1(1)m$ implies i=1, 2, 3… m.
2. Index array is initialized by n zeros.
3. compute

$$d = A^{+}b \tag{1}$$

Where $d=[\underset{\rightarrow}{d_i}]$, which is an n×1 vector and A^+ is minimum norm least square inverse.

4. Calculate

$$e = Ad \tag{2}$$

If e is not equal to b, then the LPP is inconsistent.

5. $H = A^{+}A$ (3)

$c' = (I - H)c$ (4)

$$s_k = \min\{\frac{d_i}{c'_i}; c'_i > 0\} \quad (5)$$

$$x = d - c's_k \quad (6)$$

All these have to be computed, where H is a $n \times n$ matrix, I is a $n \times n$ unit matrix, $c' = [c'_i]$ is a $n \times 1$ vector.

6. Remove the xi that becomes zero, remove the corresponding i^{th} column vector of the constraint matrix A and the corresponding element c_i from the c-vector of the objective function, shrink A and c and maintain an index counter for the variable xi that has been removed. Reduce the dimension n of A by one, n of c by one.
7. Repeat the 5th and 6th steps till s_k becomes zero.
8. The value of the objective function

$$z = c^t x \quad (7)$$

Should be then computed, where c and x are the most recent vectors.

9. Calculate the vector

$$y^t = c^t{}_B B^{-1} \quad (8)$$

10. Calculate the scalar for all non basic vectors p_j

$$Zj - Cj = y^t Pj - Cj \quad (9)$$

11. Test the sign of $Z_j - C_j$. If it is ≤ 0 then the solution is optimal otherwise it is unbounded.

All the steps specified above should be followed to get the optimal solution for the problem. Normally the linear programming problem can be written as follows

Minimize $z = c^t x$ Subject to $Ax = b$,

Where $x \geq 0$, $A = [a_{ij}]$ is a $m \times n$ constraint matrix, $c = [c_i]$ is a $n \times 1$ column vector, $b = [b_j]$ is a $m \times 1$ column vector, t is a transpose, and 0 is a null column vector.

2.1 Example

The linear programming problem in an inequality form is represented as

Min $z = -1.2x_1 - 1.4x_2$ subject to

$$40x_1 + 25x_2 \leq 1000$$
$$35x_1 + 28x_2 \leq 980$$
$$25x_1 + 35x_2 \leq 875$$
$$x_1, x_2 \geq 0$$

The linear programming problem can be written in equality constraints as follows

Min $z = c^t x$ subject to $Ax = b,\ x \geq 0,$

Where

$$c = \begin{bmatrix} -1.2 \\ -1.4 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad x = \begin{bmatrix} x1 \\ x2 \\ x3 \\ x4 \\ x5 \end{bmatrix},$$

$$A = \begin{bmatrix} 40 & 25 & 1 & 0 & 0 \\ 35 & 28 & 0 & 1 & 0 \\ 25 & 35 & 0 & 0 & 1 \end{bmatrix},$$

$$b = \begin{bmatrix} 1000 \\ 980 \\ 875 \end{bmatrix},$$

$$m = 3, n = 5.$$

The numerical vectors c, b and the numerical matrix A are the inputs according to the step 1 of the algorithm. The index array can be initialized as follows

$$indexarray = [0 \quad 0 \quad 0 \quad 0]^t.$$

Then $d = A^{+}b$ is computed as follows

$$d = \begin{bmatrix} 0.0361 & 0.0130 & -0.0361 \\ -0.0296 & -0.0035 & 0.0524 \\ 0.2966 & -0.4340 & 0.1345 \\ -0.4340 & 0.6417 & -0.2035 \\ 0.1345 & -0.2035 & 0.0682 \end{bmatrix} \begin{bmatrix} 1000 \\ 980 \\ 875 \end{bmatrix}$$

$$d = \begin{bmatrix} 17.2657 \\ 12.8164 \\ -11.0406 \\ 16.8388 \\ -5.2187 \end{bmatrix}$$

$e = Ad$ is given as $[1000 \quad 980 \quad 875]^t$ In this problem $e = b$. So the equation $Ax = b$ is consistent.

Then H is calculated as follows

$$H = A^{+} A$$

$$A = \begin{bmatrix} 40 & 25 & 1 & 0 & 0 \\ 35 & 28 & 0 & 1 & 0 \\ 25 & 35 & 0 & 0 & 1 \end{bmatrix},$$

$$A^{+} = \begin{bmatrix} 0.0361 & 0.0130 & -0.0361 \\ -0.0296 & -0.0035 & 0.0524 \\ 0.2966 & -0.4340 & 0.1345 \\ -0.4340 & 0.6417 & -0.2035 \\ 0.1345 & -0.2035 & 0.0682 \end{bmatrix}$$

$c' = (I - H)c$ is then calculated to determine sk.

$$c' = [0.0009 \quad -0.0015 \quad 0.0018 \quad 0.0107 \quad 0.0300]^t$$

By the above calculated values s_k can be determined as follows

$$s_k = \min\{\frac{d_i}{c'_i}; c'_i > 0\}$$

$$s_k = \min\left\{\frac{17.2657}{0.0009}, \frac{-11.0406}{0.0018}, \frac{16.8388}{0.0107}, \frac{-5.2187}{0.0300}\right\}$$

Here -0.0015 is neglected because it is less than 0.

$$s_k = \min\{19486, -5997, 1567, -174\}$$

Of these values -5997 is the minimum value and so the.

$$s_k = -5997$$

The value of x can be calculated as per the equation (6) as follows

$$x = d - c' s_k$$

$$d = \begin{bmatrix} 17.2657 \\ 12.8164 \\ -11.0406 \\ 16.8388 \\ -5.2187 \end{bmatrix},$$

Where

$$c' = [0.0009 \quad -0.0015 \quad 0.0018 \quad 0.0107 \quad 0.0300]^t$$

and

$$s_k = -5997$$

The value of x is given as

$$x = [22.5793 \quad 3.8732 \quad 0 \quad 81.2767 \quad 174.9568]^t$$

Remove x3 since it has become zero, i.e., non basic. Hence remove the third column vector of A and the third element of c, i.e., c3. Then shrink the 3 ×5 matrix A and the 5 ×1 vector c to the 3 ×4 matrix and 4 ×1 vector and call them once again A and c, respectively. The index array that keeps track of which element of x has become 0, i.e., non basic, now becomes $[0\ 0\ 3\ 0\ 0]^t$. Replace n by n - 1, i.e., n is now 4.

$$d = A^{+} b = [16.6990 \quad 13.2815 \quad 23.6506 \quad -7.3298]^t$$

Again H value is calculated as follows

$$H = \begin{bmatrix} 0.9991 & 0.0015 & -0.0092 & -0.0292 \\ 0.0015 & 0.9976 & 0.0148 & 0.0468 \\ -0.0092 & 0.0148 & 0.9094 & -0.2864 \\ -0,0292 & 0.0468 & -0.2864 & 0.0939 \end{bmatrix}$$

The new A^{+} is computed from the current A^{+}. Let the current matrix A is indicated as A_{K+1}.

$$A_{K+1} = \begin{bmatrix} 40 & 25 & 1 & 0 & 0 \\ 35 & 28 & 0 & 1 & 0 \\ 25 & 35 & 0 & 0 & 1 \end{bmatrix}$$

The inverse of A can be represented as A^{+}_{k+1}.

$$A^{+}{}_{k+1} = \begin{bmatrix} 0.0361 & 0.0130 & -0.0361 \\ -0.0296 & -0.0035 & 0.0524 \\ 0.2966 & -0.4340 & 0.1345 \\ -0.4340 & 0.6417 & -0.2035 \\ 0.1345 & -0.2035 & 0.0682 \end{bmatrix}$$

Let $A_k = A_{k+1}$ by leaving the third column and so remove the third column of the original matrix of A. Hence A_k is given as follows

$$A_k = \begin{bmatrix} 40 & 25 & 0 & 0 \\ 35 & 28 & 1 & 0 \\ 25 & 35 & 0 & 1 \end{bmatrix}$$

Moreover, let $A_{k+1-3} = A^{+}{}_{k+1}$ without the third row and it is given as

$$A^{+}{}_{k+1-3} = \begin{bmatrix} 0.0361 & 0.0130 & -0.0361 \\ -0.0291 & -0.0035 & 0.0524 \\ -0.4340 & 0.6417 & -0.2035 \\ 0.1345 & -0.2035 & 0.0682 \end{bmatrix}$$

Let the third column of A_{k+1} be a3 and it can be given as

$$a_3 = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^t$$

Let the third row of $A^{+}{}_{k+1}$ be $b^t{}_3$ and it is given as

$$b^t{}_3 = \begin{bmatrix} 0.2966 & -0.4340 & 0.1345 \end{bmatrix}$$

Then compute $r = 1 - b^t{}_3 a_3$. The value of r is 0.7034.

Compute

$$A^{+}{}_k = A^{+}{}_{k+1-3} + \frac{1}{r}(A^{+}{}_{k+1-3} a_3) b^t{}_3$$

Thus

$$A^{+}{}_k = \begin{bmatrix} 0.0513 & -0.0092 & -0.0292 \\ -0.0421 & 0.0148 & 0.0468 \\ -0.6170 & 0.9094 & -0.2864 \\ 0.1912 & -0.2864 & 0.0939 \end{bmatrix}$$

C', s_k, x are also calculated as follows

$$c' = \begin{bmatrix} 0.0010 & -0.0016 & 0.0096 & 0.0304 \end{bmatrix}^t$$

$$s_k = -241.1290$$

$$x = \begin{bmatrix} 16.9355 & 12.9032 & 25.9677 & 0 \end{bmatrix}^t$$

Here remove the last element of x since it has become zero. Hence remove the last column of A and the last element of c. Then shrink the 3 ×4 matrix A and the 4×1 vector c to the 3 ×3 matrix and 3 ×1 vector and call them once again A and c, respectively. The index array now becomes $[0\ 0\ 3\ 0\ 5]^t$ which means that the elements x3 and x5 have become non basic. Replace n by n-1, i.e., n is now 3.

Now again it goes to the step of $H = A^{+}A$. Here H is the unit matrix of order 3. $c' = (I - H)c$ becomes 0 and so it is null column vector. S_k is not becomes 0 and so it is null column vector. S_k is not computable for this case. Hence the current solution vector x using the information of the index array is given as

$$x = \begin{bmatrix} 16.9355 & 12.9032 & 0 & 25.9677 & 0 \end{bmatrix}^t$$

The value of the objective function is calculated as

$$z = c'x$$
$$= \begin{bmatrix} -1.2 & -1.4 & 0 & 0 & 0 \end{bmatrix} x = -38.3871$$

Next the optimality test for the solution x is carried out. For the test the basis can be given that is the original matrix A can be given without the columns 3 and 5.

$$B = \begin{bmatrix} 40 & 25 & 0 \\ 35 & 28 & 1 \\ 25 & 35 & 0 \end{bmatrix}$$

$$c^t{}_B = \begin{bmatrix} -1.2 \\ -1.4 \\ 0 \end{bmatrix}, \quad c_3 = 0$$

$$A = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$$

$$p_5 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix},$$

$p_3 = column \;\; 3 \; of \;\; original$

Since B is non singular, $B^+ = B^{-1}$ and so $y^t = c^t{}_B B^{-1}$ is calculated as

$$y^t = [-0.009 \quad 0 \quad -0.0335]$$

Then

$$z_3 - c_3 = y^t p_3 - c_3 = -0.009$$

since $c_3 = 0$

Similarly

$$z_5 - c_5 = y^t p_5 - c_5 = -0.0335$$

Here all the two $z_j - c_j$ values are negative and so the direct heuristic algorithm has given the optimal solution.

3. Conclusion

Thus the direct heuristic algorithm is used to find the optimal solution for the linear programming problems. An example shown above has explained the algorithm very briefly. The numerical examples show that the proposed algorithms give better results when compared to the existing methods and techniques.

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