IJCSN International Journal of Computer Science and Network, Volume 3, Issue 6, December 2014
ISSN (Online) : 2277-5420 **www.IJCSN.org**
Impact Factor: 0.274

A Note on Fuzzy Multi-objective Linear Fractional Programming Problem

[1] Moumita Deb, [2] P. K. De

[1, 2] Department of Mathematics, National Institute of Technology,
Silchar – 788 01, Assam, India

Abstract - The concept of ranking method is an efficient approach to rank fuzzy numbers. The aim of the paper is to find the pareto optimal solution of fuzzy multiobjective linear fractional programming (FMOLFP) problem. To study FMOLFP problem, the fuzzy coefficients and scalars in the linear fractional objectives and the fuzzy coefficients are characterised by triangular or trapezoidal fuzzy numbers. The left hand side of the fuzzy constraints are characterised by triangular or trapezoidal fuzzy numbers, while the right hand sides are assumed to be crisp number. The fuzzy coefficients and scalars in the linear fractional objectives and fuzzy coefficients in the linear constraints are transformed to crisp MOLFP problem using ranking method. The reduced problem is solved by simplex method to find the pareto optimal solution of MOLFP problem. To demonstrate the proposed approach, one numerical example is solved.

Keywords – *Fuzzy sets, Trapezoidal fuzzy number, Triangular fuzzy number, Multiobjective linear fractional programming, Ranking.*

1. Introduction

In many applications, ranking of fuzzy numbers is an important component of the decision making process. Many real-world problems require handling and evaluation of fuzzy data for making decision. To evaluate and compare different alternatives, it is necessary to rank fuzzy numbers. Multiobjective linear fractional programming problem is a very useful decision making tool. It is used for modelling real life problems with one or more objectives such as debt/equity, profit/cost, inventory/sales, actual cost/standard cost, output/employees, etc. with respect to some constraints. There are many methods to solve linear programming problem. Among all methods, simplex method is most powerful method. With the help of simplex method, linear fractional programming problem (LFPP) is also solved. This method is based on the property that the optimal solution to a linear programming problem, if it exists, can always be found in one of the basic feasible solution. The iterative steps of the simplex method are repeated until a finite optimal solution, if exists, is found. If no optimal solution, the method indicates that no finite solution exists. The concept of decision making in fuzzy environment was first studied by Bellman and Zadeh [2]. Charnes and Cooper [1] showed that the linear fractional programming problem can be optimized by solving a linear programming problem. Zimmermann [4,5] presented fuzzy approach to multi-objective linear programming problems. There are several strategies have been proposed for ranking fuzzy numbers. In 1976, Jain [3] proposed a method for ranking fuzzy numbers, then a large variety of methods have been developed to rank fuzzy numbers. Wang and Kerre [7,8] studied reasonable properties for the ordering of fuzzy quantities while Asady and Zendehnam [13] proposed a method based on distance minimization method. Abbasbandy and Hajjari [14] proposed a new method for ranking Trapezoidal fuzzy number. Nasseri et al.[12] have studied duality in fuzzy number linear programming by use of ranking function. The structure of the paper is organised as follows:

In section 2, some basic notions of the fuzzy set, different fuzzy numbers, multiobjective linear fractional programming (MOLFP) problem are defined. A new methodology of MOLFP problem using ranking methods and one algorithm has been presented to find the pareto optimal solution of the reduced MOLFP problem are discussed in section 3. In section 4, a numerical example is solved to demonstrate the algorithm. Section 5 discusses the main results and conclusions of this paper.

2. Definitions and Preliminaries

In this section, some basic definitions are given below-

Definition 2.1: If X is a collection of objects denoted generally by x, then a fuzzy set of ordered pairs:

$$\tilde{A} = \{(x, \mu_{\tilde{A}}(x)) : x \in X\}$$

where $\mu_{\tilde{A}}(x)$ is called the membership function of x in $\tilde{A}$. The membership function maps each element of X to a membership grade between 0 and 1.

Remark: In this paper we suppose that X=R.

Definition 2.2: A fuzzy number $\tilde{A}$ is a convex normalized fuzzy set on the real line R such that :
(i)It exists at least one x_0 in R with $\mu_{\tilde{A}}(x_0)$=1.

(ii) $\mu_{\tilde{A}}(x)$ is piecewise continuous.

Definition 2.3 Triangular Fuzzy Numbers:
Let $\tilde{A}$=(a,b,c), a < b < c be a fuzzy set on R=(-∞, ∞). It is called a triangular fuzzy number, if its membership function is

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{x-a}{b-a}, if\ a \le x \le b \\ \frac{c-x}{c-b}, if\ b \le x \le c \\ 0\ , otherwise \end{cases}$$

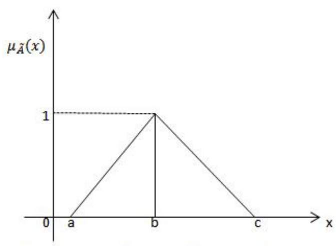

Figure 1: Triangular Fuzzy Number $\tilde{A}$=(a, b, c)

If a=b=c then $\tilde{A}$=(a, a, a) .
Let F denote the all triangular fuzzy numbers.

2.3.1 Arithmetic Operations of Triangular Fuzzy Number:
Let $\tilde{A}$ and $\tilde{B}$ are two triangular fuzzy number of LR-type:
$\tilde{A} = (a_1, a_2, a_3)$, $\tilde{B} = (b_1, b_2, b_3)$ then

1. $\tilde{A} + \tilde{B} = (a_1, a_2, a_3) + (b_1, b_2, b_3) = (a_1 + b_1, a_2 + b_2, a_3 + b_3)$
2. $\tilde{A} - \tilde{B} = (a_1, a_2, a_3) - (b_1, b_2, b_3) = (a_1 + b_3, a_2 - b_2, a_3 + b_1)$
3. $\tilde{A} * \tilde{B} = (a_1, a_2, a_3) * (b_1, b_2, b_3) = (a_1 b_2 + a_2 b_1, a_2 b_2, a_2 b_3 + a_3 b_2)$
4. $\tilde{A} / \tilde{B} = (a_1, a_2, a_3) / (b_1, b_2, b_3) = (a_1 / b_2 + a_2 / b_1, a_2 / b_2, a_2 / b_3 + a_3 / b_2)$

Definition 2.4 Trapezoidal Fuzzy Number:
A fuzzy number $\tilde{A}$=(a,b,c,d), $a \le b \le c \le d$ (a,b,c,d > 0) is said to be a trapezoidal fuzzy number if its membership function is given by-

$$\mu_{\tilde{A}}(x) = \begin{cases} 0 & , if\ \ x < a \\ \frac{x-a}{b-a} & , if\ \ a < x \le b \\ 1 & , if\ b < x < c \\ \frac{d-x}{d-c} & , if\ c \le x < d \\ 0 & , if\ x > d \end{cases}$$

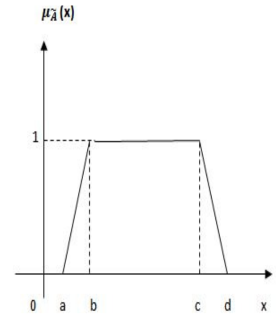

Figure 2: Trapezoidal Fuzzy Number $\tilde{A}$=(a, b, c, d)

Remark: If b=c, the trapezoidal fuzzy number reduces to a triangular fuzzy number as shown in figure3. It can be written by a triplet (a,b,d).

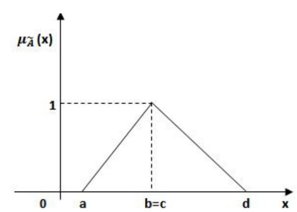

Figure 3: Triangular Fuzzy Number $\tilde{A}$=(a, b, c)

2.4.1 Arithmetic Operations of Trapezoidal Fuzzy Number:

Function principle is proposed by Chen [6] to be as the fuzzy arithmetical operations by trapezoidal fuzzy numbers. This method is more useful then the extension principle [11] for the fuzzy numbers with the trapezoidal membership function.

We describe some fuzzy arithmetical operations under function principle as follows:-
Suppose, $\tilde{A} = (a_1, a_2, a_3, a_4)$ and $\tilde{B} = (b_1, b_2, b_3, b_4)$ are two trapezoidal fuzzy numbers. Then-

1. $\tilde{A} \oplus \tilde{B} = (a_1 + b_1, a_2 + b_{2,} a_3 + b_3, a_4 + b_4)$,

2. $\tilde{A} \otimes \tilde{B} = (a_1 b_1, a_2 b_{2,} a_3 b_3, a_4 b_4)$,

3. $-\tilde{B} = -(b_1, b_2, b_3, b_4) = (-b_4, -b_3, -b_2, -b_1)$,

4. $\tilde{A} \ominus \tilde{B} = (a_1 - b_4, a_2 - b_3, a_4 - b_2, a_4 - b_1)$,

5. $\frac{1}{\tilde{B}} = \widetilde{B^{-1}} = (\frac{1}{b_4}, \frac{1}{b_3}, \frac{1}{b_2}, \frac{1}{b_1})$,

6. $\tilde{A} \oslash \tilde{B} == (\frac{a_1}{b_4}, \frac{a_2}{b_3}, \frac{a_3}{b_2}, \frac{a_4}{b_1})$,

7. Let, $\lambda \in R$,
   For $\lambda > 0, \lambda \otimes \tilde{A} = (\lambda a_1, \lambda a_2, \lambda a_3, \lambda a_4)$,
   For $\lambda < 0, \lambda \otimes \tilde{A} = (\lambda a_4, \lambda a_3, \lambda a_2, \lambda a_1)$

where $a_1, a_2, a_3, a_4, b_1, b_2, b_3$ *and* b_4 are all non-zero positive real numbers.

\

IJCSN International Journal of Computer Science and Network, Volume 3, Issue 6, December 2014
ISSN (Online) : 2277-5420 **www.IJCSN.org**
Impact Factor: 0.274

Definition 2.5 Multiobjective Linear Fractional Programming Problem:
The general format of a multiobjective linear fractional programming problem which is stated as follows-

Max Z(x)={$Z_1(x), Z_2(x), \ldots\ldots.., Z_p$}

subject to the constraints:

$$x \in \Delta=\{x\in R^n:\ Ax\begin{pmatrix}\le\\=\\\ge\end{pmatrix}b,\ x\ge 0\}$$

with b∈ R^n and A∈ $R^{m\times n}$

and $Z_p(x) = \frac{c_p x + \alpha_p}{d_p x + \beta_p} = \frac{N_p(x)}{D_p(x)}$;

where $c_p, d_p \in R^n$ and $\alpha_p, \beta_p \in R$.

3. Methodology

The fuzzy multiobjective linear fractional programming problem is of the following form:

$$\text{Maximize } \frac{\tilde{c}_{r1}x_1+\tilde{c}_{r2}x_2+\cdots\ldots\ldots+\tilde{c}_{rn}x_n+\tilde{c}_{r0}}{\tilde{d}_{r1}x_1+\tilde{d}_{r2}x_2+\cdots\ldots\ldots+\tilde{d}_{rn}x_n+\tilde{d}_{r0}}$$

r=1,2,………,p

subject to the constraints:

$$\sum_{j=1}^{n}\tilde{a}_{ij} \le b_i,\ \text{i=1,2,…..,m} \qquad (1)$$
$$x_j \ge 0,\ \text{j=1,2,…….,n}$$

where x_j are non-negative decisions variables, $\tilde{c}_{rj}$ and $\tilde{d}_{rj}$, j=1, 2,....., n are fuzzy coefficients, while $\tilde{c}_{r0}$ and $\tilde{d}_{r0}$ are fuzzy scalars, for the r^{th} linear fractional objective function, r=1, 2,......, p and p is the number of the distinct fuzzy linear fractional objective functions . b_i, i=1, 2 ,..., m are independent random variables with known distribution functions, while $\tilde{a}_{ij}$ represents the fuzzy coefficient of the j^{th} decision variable in the i^{th} stochastic constraint.

By using Charnes-Cooper's variable transformation [1], the multiobjective fuzzy linear programming model which is equivalent to the model (1) can be written as:

$$\text{Maximize } f_j(y_j,t) = \sum_{j=1}^{n}\tilde{c}_{rj}y_j + \tilde{c}_{r0}t,$$

subject to:

$$\sum_{j=1}^{n}\tilde{a}_{ij}y_j - b_i t \le 0,$$
$$\sum_{j=1}^{n}\tilde{d}_{rj}y_j + \tilde{d}_{r0}t = 1, \qquad (2)$$

$$y_j, t \ge 0 \text{ and } y_j = t\, x_j$$

$$\text{where } t=\frac{1}{\tilde{d}_{r1}x_1+\tilde{d}_{r2}x_2+\cdots\ldots\ldots+\tilde{d}_{rn}x_n+\tilde{d}_{r0}},$$

r=1,2,........,p; i =1,2,.......,m; j=1,2,......,n.

3.1 Ranking of Fuzzy Numbers

There are many methods are available for ranking fuzzy numbers. An efficient approach for ordering the fuzzy numbers is defined by a ranking function d:F(R)→ R which maps for each fuzzy number into the real line, where a natural order exists. We define orders on F(R) as follows:

(i) $\tilde{A} \ge \tilde{B}$ iff d($\tilde{A}$)≥ $d(\tilde{B})$,

(ii) $\tilde{A} \le \tilde{B}$ iff d($\tilde{A}$)≤ $d(\tilde{B})$,

(iii) $\tilde{A} = \tilde{B}$ iff d($\tilde{A}$)= $d(\tilde{B})$.

3.1.1 Ranking of Triangular Fuzzy Number

Chen and Cheng [10] proposed a metric distance method to rank fuzzy numbers .
Let $\tilde{A} = (a_1, a_2, a_3)$ be a triangular fuzzy number of LR-type then R($\tilde{A}$)=$\frac{a_1+2a_2+a_3}{4}$.

3.1.2 Ranking of Trapezoidal Fuzzy Number:[11]

Let, $\tilde{B} = (b_1, b_2, b_3, b_4)$be a trapezoidal fuzzy number .Then by graded mean integration representation of $\tilde{B}$can be written as:

$$R(\tilde{B})=\frac{b_1+2b_2+2b_3+b_4}{6}.$$

3.2 Algorithm for solving Fuzzy Multiobjective Linear Fractional Programming Problem

In this paper, we use ranking approach in FMOLFP model to reduce it in MOLFP model. The ranking approach algorithms for FMOLFP model are explained as follows:

Step 1: Convert the FMOLFP model (1) to MOLFP model by using ranking approach where fuzzy coefficients and scalars are characterized by trapezoidal fuzzy number or triangular fuzzy number.

Step 2: For each objective function, convert each fuzzy coefficients and scalars which are in trapezoidal fuzzy number to crisp number by using graded mean integration representation (GMIR) method while that of triangular fuzzy number to crisp number by using metric distance (MD) method.

Step 3: For each constraint, convert LHS of each fuzzy coefficients which are in trapezoidal fuzzy number to crisp number by GMIR method and that of triangular fuzzy number to crisp number by MD method.

Step 4: Assume RHS of each fuzzy constraints as crisp number.

\

Step5: Substitute $y_j = t\,x_j$ in the model (1) where t=$\frac{1}{\tilde{d}_{r1}x_1+\tilde{d}_{r2}x_2+\cdots\cdots+\tilde{d}_{rn}x_n+\tilde{d}_{r0}}$ and $y_j, t \geq 0$, i=1,2,......,m; j=1,2,......,n; r=1,2,........,p,

Step 6: Solve model (2) to get the values of $y_1, y_2, \ldots\ldots\ldots, y_n$,t.

Step 7: Convert the above values $y_1, y_2, \ldots\ldots\ldots, y_n$, by using the transformation $y_j = t\,x_j$, j=1,2,.......,n.

Step 8: Compute the values of $z_1, z_2, \ldots\ldots\ldots, z_n$, by using the values of $x_1, x_2, \ldots\ldots\ldots, x_n$.

4. Numerical Example

Example 1[9]: To demonstrate the feasibility of the proposed approach, consider the fuzzy multiobjective linear fractional programming problem-

Maximize
$$\frac{(4,7,10,12)x_1+(8,10,14,15)x_2+(2.5,4,7.5,11.5)x_3+(2,3,4,6)}{(10,14,20,22)x_1+(20,23.5,27.5,29)x_2+(18,20,25,28)x_3+(5,10,18,20)}$$
Maximize
$$\frac{(20,24,28)x_1+(18,25,30)x_2+(14,19,25)x_3+(1,6,10)}{(14,16,19,23)x_1+(18,21,25,27)x_2+(15,20,25,30)x_3+(10,15,20,25)}$$

Subject to the constraints:

$(10,17,19,25)x_1 + (14,16,22,24)x_2 + (20,25,27,30)x_3 \leq 44.96$,
$(0.03,0.07,0.09)x_1 + (0.05,0.08,0.1)x_2 + (0.02,0.06,0.07)x_3 \leq 0.9163$,
$(4,6,10,13)x_1 + (0,5,10,15)x_2 + (8,11,14,20)x_3 \leq 35$,
$x_1, x_2, x_3 \geq 0$.

Solution:
The above FMOLFPP is equivalent to the following MOLFPP:

Maximize $\frac{8.3x_1+11.8x_2+6.16x_3+3.6}{16.6x_1+25.1x_2+22.6x_3+13.5}$

Maximize $\frac{24x_1+24.5x_2+19.2x_3+5.75}{17.8x_1+22.8x_2+22.5x_3+17.5}$

subject to the constraints:

$17.8x_1 + 19x_2 + 25.6x_3 \leq 44.96$,
$0.065x_1 + 0.0775x_2 + 0.0525x_3 \leq 0.9163$,
$8.16x_1 + 7.5x_2 + 13x_3 \leq 35$,
$x_1, x_2, x_3 \geq 0$.

Here substituting $y_1 = tx_1, y_2 = tx_2, y_3 = tx_3$ and

t=$\frac{1}{16.6x_1+25.1x_2+22.6x_3+13.5}$, t=$\frac{1}{17.8x_1+22.8x_2+22.5x_3+17.5}$

The above MOLFPP is reduced to MOLPP as follows:

Maximize $f_1(y,t)= 8.3y_1 + 11.8y_2 + 6.16y_3 + 3.6t$,
Maximize $f_2(y,t) = 24y_1 + 24.5y_2 + 19.2y_3 + 5.75t$,
subject to the constraints:
$17.8y_1 + 19y_2 + 25.6y_3 - 44.96t \leq 0$,
$0.065y_1 + 0.0775y_2 + 0.0525y_3 - 0.9163t \leq 0$,
$8.16y_1 + 7.5y_2 + 13y_3 - 35t \leq 0$,
$16.6y_1 + 25.1y_2 + 22.6y_3 + 13.5$t=1,
$17.8y_1 + 22.8y_2 + 22.5y_3 + 17.5$t=1,

where t, $y_1, y_2, y_3 \geq 0$.

Solving by LP package [15], we get,
y_1=0.01 ,y_2=0.03 , y_3=0 and t=0.01.
y_1=0.01 ,y_2=0.03 , y_3=0 and t=0.01.
Therefore $f_1(y,t)$=0.36 and $f_2(y,t)$=1.03

Hence the pareto optimal solution of the problem are Z_1(x)=0.43 and Z_2(x)=0.92.

5. Conclusions

In this paper, fuzzy multiobjective linear fractional programming (FMOLFP) problem is solved by using ranking method. Here, we have applied ranking method of triangular and trapezoidal fuzzy number in FMOLFP problem. The fuzzy coefficients and scalars of both objective functions and linear constraints are transformed to crisp number by using metric distance ranking (in triangular fuzzy number) and graded mean integration representation method (in trapezoidal fuzzy number). The reduced problem has been solved by using standard LP package. After solving the problem, the values of objective functions are obtained by using different arithmetic operations of triangular or trapezoidal fuzzy number.
One numerical example is presented to demonstrate our approach. The proposed approach to solve FMOLFP problem yields an efficient solution which reduces the complexity in problem solving and it is easy to compute. Finally, we see that the values of second objective function gives better pareto optimal solution than first objective function. This type of problems are applicable to different areas like financial sector, inventory management, production planning, banking sector etc.

References

[1] A. Charnes and W.W.Cooper, "Programming with linear fractional functional", Naval Research Logistics Quart., 9, 1962, 181-186.
[2] R.E. Bellman and L.A. Zadeh, "Decision making in fuzzy environment", Management Science, 17, 1970, 141-146.
[3] R. Jain, "Decision making in the presence of fuzzy variable", IEEE Transactions on Systems Man, and Cybernetics ,6, 1976, 698-703.
[4] H.-J. Zimmermann, "Fuzzy programming and linear programming with several objective functions", Fuzzy Sets and Systems, 1,1978, 45-55.
[5] H.-J.Zimmermann, "Fuzzy mathematical programming", Computers and Operations Research, Vol.10, 1983, 291-298.
[6] S.H.Chen, "Operations of fuzzy numbers with function principle", Tamkang Journal of Management Sciences, 6/1, 1985, 13-26.

[7] X. Wang and E.E. Kerre, "Reasonable properties for the ordering of fuzzy quantities(I)", Fuzzy Sets and Systems, 118, 2001a, 375-385.

[8] X. Wang and E.E. Kerre, "Reasonable properties for the ordering of fuzzy quantities(I)", Fuzzy Sets and Systems, 118, 2001b, 387-405.

[9] M.G. Iskander, "A Possibility programming approach for stochastic fuzzy multiobjective linear fractional programs", Computers and Mathematics with Applications, 48, 2004, 1603-1609.

[10] LS Chen and CH Cheng, "Selecting IS personnel using ranking fuzzy number by metric distance method", European Journal of Operational Research ,160(3), 2005, 803-820.

[11] S.H.Chen, S.T.Wang and S.M.Chang, "Some properties of Graded mean integration representation of LR type fuzzy numbers" , Tamsui Oxford Journal of Mathematical Sciences, Aletheia University, 22(2), 2006, 185-208.

[12] N. Mahdavi-Amiri and S.H.Nasseri, "Duality in fuzzy number linear programming by use of a certain linear ranking function", Applied Mathematics and Computation, 180, 2006, 206-216.

[13] B. Asady and A. Zendehnam, "Ranking fuzzy numbers by distance minimizing", Applied Mathematical Modeling ,31, 2007, 2589-2598.

[14] S. Abbasbandy and T. Hajjari, "A new approach for ranking of Trapezoidal fuzzy numbers" Computers and Mathematics with Applications, 57, 2009, 413-419.

[15] H.M.Taha, "Operations Research- An Introduction with AMPL, Solver, Excel, and Tora Implementations, PEARSON, Printice Hall, 8^{th}-edition, New Jersey 07458.

Ms. Moumita Deb completed her Masters degree in Mathematics from Assam University, Silchar in the year 1998. In 2000, she obtained B.Ed. degree from Assam University, Silchar. Currently she is working as a Research scholar in Department of Mathematics, NIT Silchar since 2011. Her research interests includes linear fractional programming problem and multiobjective linear fractional programming problem.

Dr. Pijus Kanti De obtained his M.Sc in Mathematics and B.T teaching training degrees from Kalyani University . Also he obtained his M.Phil and Ph.D in Applied Mathematics degrees from Indian School of Mines Dhanbad. Presently he is working as an Associate Professor in Mathematics in National Institute of Technology Silchar since 2010. Previously Dr.De was in Banasthali University (Raj), KIET Ghaziabad under U.P.T.U, Delhi College of Engineering (D.T.U) and Centre for Mathematical Modeling and Computer Simulation (C-MMACS), National Aerospace Laboratories (NAL), Bangalore. His research interest lies on Fuzzy Optimization and Decision Making, Operations Research, Fuzzy Mathematics, Mathematical Modeling , Uncertainty Modeling, Numerical Optimization, Numerical Analysis, Elasto-Dynamics, Wave Propagation and Applied Mathematics.