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A Review On Nano Quad Topological Spaces

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Abstract - In this paper, A new kind of topology is called Nano Quad topology and is denoted by N-Q. N-Q- opensets, N-Q-Pre open, N-Q-Semi open are introduced and also studied continuity of Nano Quad topology.

Keywords - *NanoQuad Topological spaces, N-Q- open sets, N-Q- interior, N-Q- closure , N-Q- Pre opensets, N-Q- semi open sets*

1. Introduction

J.C. Kelly [7] introduced bitopological spaces in 1963. The study of tri-topological spaces was first initiated by Martin M. Kovar [3] in 2000,where a non empty set X with three topologies is called tri-topolgical spaces. I.N.F. Hameed and Moh. Yahya Abid [I] gives the definition of 123 open set in tri topological spaces . In 2014 Palaniammal[9] and Somasundaram introduced a topology $T_1 \cap T_2 \cap T_3$ in the tri topological space (X, T_1, T_2, T_3) and studied several properties of this topology. Stella Irene Mary J introduce a new topology called Tri star topology induced by two bitopology and is denoted by N-Q. D.V. Mukundan [4] introduced the concept on topological structures with four topologies, quad topology (4 – tuple topology) and defined new types of open (closed)sets. The concept of nanotopology was introduced by Lellis Thivagar [14]. K.Buvaneshwari[15] etal S.Chandrasekar[12] etal contributed in Nanobitopological spaces. S.Chandrasekar[16] introduce a new topology called NanoTri star topology induced by two nano bitopology and is denoted by $NT^*_{123.}$

In this paper, we introduce a new topology called Nano Quad topology and is denoted by N-Q. The various concepts of N-Q interior, N-Q closure, N-Q Preopen sets, N-Q semi open sets and N-Q continuous are analyzed.

2. Preliminaries

Definition 2.1 [6]
A topology on a non empty set X is a collection T of subsets of X having the following the properties:

1) X and ϕ are in T.
2) The union of the elements of any sub collection of T is in T.
3) The intersection of the elements of any finite sub collection of T is in T.

A set X for which a topology T has been specified is called a Topological space.

Definition 2.2.[14]
Let U be a non-empty finite set of objects called the universe and R be an equivalence relation on U named as the indiscernibility relation. Elements belonging to the same equivalence class are said to be indiscernible with one another. The pair (U, R) is said to be the approximation space. Let X⊆U.
(i) The lower approximation of X with respect to R is the set of all objects, which can be for certain classified as X with respect to R and it is denoted by $L_R(X)$ .That is, $L_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \subseteq X\}$ where R(x) denotes the equivalence class determined by X.
(ii) The upper approximation of X with respect to R is the set of all objects, which can be possibly classified as X with respect to R and it is denoted by $U_R(X)$.That is, $U_R(X) = \bigcup_{x \in U}\{R(x) : R(x) \cap X = \phi\}$(iii) The boundary region of X with respect to R is the set of all objects, which can be neither in nor as not-X with respect to R and it is denoted by $B_R(X)$. That is, $B_R(X) = U_R(X) - L_R(X)$

Definition 2.3 [14]
If (U, R) is an approximation space and X, Y ⊆ U, then
(i) $L_R(X) \subseteq X \subseteq U_R(X)$

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(ii) $L_R(\phi) = U_R(\phi) = \phi$
(iii) $U_R(X \cup Y) = U_R(X) \cup U_R(Y)$
(iv) $U_R(X \cap Y) \subseteq U_R(X) \cap U_R(Y)$
(v) $L_R(X \cup Y) \supseteq L_R(X) \cup L_R(Y)$
(vi) $L_R(X \cap Y) = L_R(X) \cap L_R(Y)$
(vii) $L_R(X) \subseteq L_R(Y)$ and $U_R(X) \subseteq U_R(Y)$ whenever $X \subseteq Y$
(viii) $U_R(X^c) = [L_R(X)]^c$ and $L_R(X^c) = [U_R(X)]^c$
(ix) $U_R\, U_R(X) = L_R\, U_R(X) = U_R(X)$
(x) $L_R\, L_R(X) = U_R\, L_R(X) = L_R(X)$

Definition2.4.[14]
Let U be an universe, R be an equivalence relation on U and $\tau_R(X) = \{U, \phi, L_R(X), U_R(X), B_R(X)\}$ where $X \subseteq Y$. $\tau_R(X)$ satisfies the following axioms
(i) $U, \phi \in \tau_R(X)$
(ii)The union of the elements of any sub-collection of $\tau_R(X)$ is in $\tau_R(X)$
(iii)The intersection of the elements of any finite sub-collection of $\tau_R(X)$ is in $\tau_R(X)$.
That is, $\tau_R(X)$ forms a topology on U called the nano topology on U with respect to X. We call(U, $\tau_R(X)$)) as the nano topological space. The elements of $\tau_R(X)$ are called nano open sets.

3. Nano Quad Topological Space

Definition 3.1.
Let U be a nonempty set and Let $\tau_{R_1}(X)$, $\tau_{R_2}(X)$, $\tau_{R_3}(X)$and $\tau_{R_4}(X)$are Nano topology on U. Then a subset A of space is said to be Nano Quad-open (N-Q-open) set if $A \in [\tau_{R_1}(X) \cup \tau_{R_2}(X) \cup \tau_{R_3}(X) \cup \tau_4(X)]$ and complement is said to be Q-closed set and the set with four topologies called $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$Nano Quad topology (N-Q-topology). Obviously by definition of N-Q-open sets it satisfies all the axioms of Nano topology U

Example:3.2
Let U = {p, q , r, s, t}, U/R_1= { {p} , {q , r, s}, {t}}.
LetX_1={p,q}⊆U.Then$\tau_{R_1}(X)$={U, ϕ ,{p},{p,q,r,s},{q,r,s}}
LetX_2={p,r}⊆U.The$\tau_{R_2}(X)$={U, ϕ ,{p},{p,q,r,s},{q,r,s}}.
Let X_3 = {q , r}⊆ U. Then $\tau_{R_3}(X)$={U, ϕ ,{q,r,s} }.
Let X_4 = {q , s}⊆ U. Then $\tau_{R_4}(X)$={U, ϕ ,{q,r,s} }.
N-Q- O$(X) = [\tau_{R_1}(X) \cup \tau_{R_2}(X) \cup \tau_{R_3}(X) \cup \tau_4(X)]$
Then N-Q- O(X)= {U, ϕ , {p},{p,q,r,s},{q,r,s}}
N-Q- C(X)= {U, ϕ , {t},{p,t},{q,r,s,t}}

Definition 3.3.
$A \subset \left(U, \tau_{R_{1,2,3,4}}(X)\right)$ is called N-Q-open in U, The union of all N-Q-open sets contained in A is called the N-Q-interior of A and denoted by N-Q-int A. We say A is N-Q-closed in U if A^C is N-Q-open, and the intersection of N-Q-closed sets containing A is called N-Q-closure of A and it is denoted by N-Q-cl(A) .

Definition 3.4.[9]
Let (X, T) be a topological space. A⊂X is called
1. Semi-open if A⊆cl(int(A)) and Semi-closed set if int(cl(A))⊆A.
2. Pre-open if A⊆int(cl(A)) and Pre-closed set if cl(int(A))⊆A.

4. N-Q Pre Open Sets:

Definition 4.1
Let $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$ be a N-Q-topological space.
A subset A of $A \subset \left(U, \tau_{R_{1,2,3,4}}(X)\right)$is called N-Q-pre open in U, if A⊆N-Q-int(N-Q-cl A).
The complement of N-Q-pre open set is called N-Q-pre closed. i.e., N-Q-cl(N-Q-int A)⊆A.

Example 4.2.
Let U = {p, q , r, s, t}, U/R_1= { {p} , {q , r, s}, {t}}.
LetX_1={p,q}⊆U.Then$\tau_{R_1}(X)$={U, ϕ ,{p},{p,q,r,s},{q,r,s}}
LetX_2={p,r}⊆U.Then$\tau_{R_2}(X)$={U ϕ,{p},{p,q,r,s},{q,r,s }}.
Let X_3 = {q , r}⊆ U. Then $\tau_{R_3}(X)$={U, ϕ ,{q,r,s} }.
Let X_4 = {q , s}⊆ U. Then $\tau_{R_4}(X)$={U, ϕ ,{q,r,s} }.
Then N-Q- O(X)= {U, ϕ , {p},{p,q,r,s},{q,r,s}}
Clearly A = { q, r, s } is N-Q -pre open.

Theorem 4.3.
Every N-Q-open set is N-Q-pre open.

Proof:
Let A be N-Q-open. Then A⊆N-Q-int(N-Q-int A). Since N-Q -int(N-Q -int A)⊆ N-Q -int(N-Q -cl A), it follows that A⊆N-Q -int(N-Q -cl A). Hence A is N-Q-pre open.

Remark 4.4
Converse of the above Theorem need not be true.

Example 4.5.
Let U = {i, j, k, l, m}U/R_1= { {i} , {j , k, l}, {m}}.
Let X_1 ={i , j}⊆ U.Then $\tau_{R_1}(X)$={U, ϕ ,{i},{i, j, k, l}, {j ,k, l}}
Let X_2 ={i ,k}⊆ U.Then $\tau_{R_2}(X)$={U, ϕ ,{i},{i, j, k ,l} , {j, k, l }}.
Let X_3 = {j , k}⊆ U. Then $\tau_{R_3}(X)$={U, ϕ ,{j, k ,l} }.
Let X_4 = {j , k}⊆ U. Then $\tau_{R_4}(X)$={U, ϕ ,{j, k ,l} }.
Then N-Q- O(X)= {U, ϕ , {i},{ i , j, k ,l},{j, k, l}}

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Clearly A = { i, j, k, m} is N-Q -pre open but not N-Q-open.

Theorem 4.6

i) Arbitrary union of N-Q- pre open sets is N-Q-pre open.

ii) Arbitrary intersection of N-Q- pre closed sets is N-Q- pre closed.

Proof:

i) Let $\{A_\alpha \mid \alpha \in I\}$ be the family of N-Q-pre open sets in X. By Definition 3.2.1, for each α, $A_\alpha \subseteq$ N-Q-int(N-Q-cl(A_α)), this implies that $\cup A_\alpha \subseteq \cup$(N-Q-int(N-Q-cl($A_\alpha$)). Since $\cup$(N-Q-int(N-Q-cl(A_α))$\subseteq$N-Q-int ($\cup$N-Q-cl(A_α)) and N-Q-int ($\cup$N-Q-cl(A_α)) = N-Q-int (N-Q-cl ($\cup A_\alpha$)), this implies that $\cup A_\alpha \subseteq$ N-Q-int (N-Q-cl($\cup A_\alpha$)). Hence $\cup A_\alpha$ is N-Q-pre open. ii) Let {Bα | $\alpha \in$I } be a family of N-Q-pre closed sets in X. Let $A_\alpha = B_\alpha^C$, then $\{A_\alpha / \alpha \in I\}$ is a family of N-Q-pre open sets. By (i), $\cup A_\alpha = \cup B A_\alpha^C$ is N-Q-pre open. Consequently $(\cap B\alpha)^C$ is N-Q-pre open. Hence ($\cap$Bα) is N-Q-pre closed.

Remark 4.7

Finite intersection of N-Q- pre open sets need not be N-Q-pre open.

Example 4.8.

In Example 4.5 {i, *l*} and {j, *l*} are N-Q- pre open sets, but {i, *l*}$\cap${j, *l*}={*l*} is not N-Q- pre open.

Theorem 4.9.

In a N-Q topological space $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$the set of all N-Q- pre open sets form a generalized topology.

Proof:

Proof follows from Remark 4.4, Theorem 4.3, Theorem 4.6 (i) and Remark 4.7.

Definition 4.10

Let $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$ be a N-Q -topological space. An element x $\in$ A is called N-Q- pre interior point of A, if there exist a N-Q- pre open set H such that x $\in$ H$\subset$A.

Definition 4.11

The set of all N-Q-pre interior points of A is called the N-Q- pre interior of A, and is denoted by N-Q- pre-int(A).

Theorem 4.12

i) Let A$\subset\left(U, \tau_{R_{1,2,3,4}}(X)\right)$Then N-Q- pre int A is equal to the union of all N-Q- pre open set contained in A.

ii) If A is a N-Q- pre open set then A= N-Q- pre int A.

Proof:

i) We need to prove that, N-Q- pre int A= $\cup${B | B$\subset$A, B is N-Q- pre open set}. Let x $\in$ N-Q- pre int A. Then there exist a N-Q- pre open set B such that x $\in$ B $\subset$ A. Hence x $\in$ $\cup${B | B$\subset$A, B is N-Q- pre open set}. Conversely, suppose x $\in$ $\cup${ B | B$\subset$A, B is N-Q- pre open set}, then there exist a set $B_o \subset$ A such that x $\in$ Bo, where B_o is N-Q- pre open set. i.e., x $\in$ N-Q- pre int A. Hence $\cup${B | B$\subset$A, B is N-Q- pre open set} $\subset$N-Q- pre int A. So N- Q- pre int A= $\cup${B| B$\subset$A, B is N-Q- pre open set}.

ii) Assume A is a N-Q- pre open set then A $\in$ {B|B$\subset$ A, N-Q- pre open set}, and every other element in this collection is subset of A. Hence by part (i) N-Q- pre int A=A.

Note 4.13:

1. N-Q- pre int A is N-Q- preopen.
2. N-Q- pre int A is the largest N-Q-pre open set contained in A.

Theorem 4.14:

i) N-Q- pre int (A$\cup$B)$\supset$N-Q- pre int A$\cup$N-Q- pre int B.

ii) N-Q- pre int (A$\cap$B) = N-Q- pre int A $\cap$N-Q - pre int B.

Proof:

i) The fact that N-Q- pre int A$\subset$A and N-Q- pre int B$\subset$B implies N-Q- pre int A$\cup$N-Q- pre int B$\subset$ A$\cup$B. Since pre interior of a set is pre open, N-Q-pre int A and N-Q-pre int B are pre open. Hence by Theorem 4.6 of (i), N-Q-pre int A$\cup$N-Q- pre int B is pre open and contained in A$\cup$B. Since N-Q- pre int (A$\cup$B) is the largest N-Q-pre open set contained in A$\cup$B, it follows that N-Q- pre int A$\cup$N-Q- pre int B $\subset$N-Q- pre int (A$\cup$B).

ii) Let x $\in$ N-Q- pre int (A$\cap$B). Then there exist a N-Q-pre open set H, such that x $\in$ H$\subset$(A$\cap$B). That is there exist a N-Q-pre open set, such that x $\in$ H$\subset$A and x $\in$ H$\subset$B. Hence x $\in$ N-Q- pre int A and x $\in$ N-Q- pre int B. That is x $\in$ N-Q- pre int A$\cap$N-Q- pre int B. Thus N-Q- pre int (A$\cap$B)$\subset$N-Q- pre int A $\cap$N-Q- pre int B. Retracing the above steps, we get the converse.

5. N-Q –Pre Closed Sets

Definition 5.1:

$\left(U, \tau_{R_{1,2,3,4}}(X)\right)$be a N-Q-topological space. Let A$\subset$ X. The intersection of all N-Q- pre closed sets containing A is called N-Q- pre closure of A and it is denoted by N-Q- pre cl(A). N-Q- pre cl(A) = $\cap${B / B$\supset$A, B is N-Q- pre closed set}.

Note 5.2:

1. N-Q- pre cl(A) is also a N-Q- pre closed set.
2. N-Q-precl(A)is smallestN-Q-preclosed setcontaining A.

Theorem 5.3:

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Every N-Q-closed set is N-Q-pre closed.

Proof:

Let A be N-Q-closed, then by Theorem 4.6, we haveN-Q-cl(N-Q-cl A)⊆A. Since N-Q-cl(N-Q-int A)⊆N-Q-cl(N-Q-cl A)⊆A, A is N-Q-pre closed.

Remark 5.4:

Converse of the above Theorem need not be true.

Example 5.5:

U = {1, 2, 3, 4}, with U/R= {{1},{3}, {2, 4}} and

X_1= {1, 2}⊆ U $\tau_{R_1}(X)$= {U, ϕ, {1}, {1, 2, 4}, {2, 4}}

X_2= {1, 4}⊆ U $\tau_{R_2}(X)$= {U, ϕ, {1}, {1, 2, 4}, {2, 4}}

X_3= {2, 4}⊆ U $\tau_{R_3}(X)$= {U, ϕ, {2, 4}}

X_4= {1,2, 4}⊆ U $\tau_{R_4}(X)$= {U, ϕ, {1,2, 4}}

N-Q- C(X)= { U, ϕ, {3},{1,3}, {2,3, 4}}

Then A = {1,2,3} is N-Q-pre closed but not N-Q-closed.

Theorem 4.6:

A is N-Q- pre closed if and only if A = N-Q- pre cl(A).

Proof:

N-Q- pre cl(A) = ∩ {B/B ⊃ A, B is N-Q- pre closed set}. If A is a N-Q- pre closed set then A is a member of the above collection and each member contains A. Hence their intersection is A and N-Q- pre cl(A) = A. Conversely, if A= N-Q- pre cl(A), then A is N-Q- pre closed by Note 4.2.

6. N-Q-Semi Open Sets

Definition 6.1

Let $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$be a N-Q- topological space. A subset A of $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$is called N-Q-semi open in X, if A ⊆ N-Q-cl(N-Q-int A). The complement of N-Q-semi open set is called N-Q-semi closed. i.e., N-Q-int(N-Q-cl A)⊆ A.

Example 6.2:

U = {a, b, c, d}, with U/R= {{a},{c}, {b, d}} and

X_1= {a, b}⊆ U, $\tau_{R_1}(X)$= {U, ϕ, {a}, {a, b, d}, {b, d}}

X_2= {a, d}⊆ U ,$\tau_{R_2}(X)$= {U, ϕ, {a}, {a, b, d}, {b, d}}

X_3= {b, d}⊆ U , $\tau_{R_3}(X)$= {U, ϕ, {b, d}}

X_4= {a,b, d}⊆ U ,$\tau_{R_4}(X)$= {U, ϕ, {a,b, d}}

Then N-Q-O(X)={U, ϕ, {a}, {a, b, d}, {b, d}}

Then N-Q- C(X)= { U, ϕ, {c},{a,c}, {b,c, d}}

Clearly A = {b,d} is N-Q-semi open.

Theorem 6.3:

i) Every N-Q-open set is N-Q-semi open.

ii) Every N-Q-closed set is N-Q-semi closed.

Proof:

i) If A is N-Q-open set then by then, A⊆N-Q-int(N-Q-int A). Since N-Q-int(N-Q-int A)⊆N-Q-cl(N-Q-int A), A⊆ N-Q-cl(N-Q-int A). Hence A is N-Q-semi open.

ii) If A is N-Q-closed set then by Theorem 4.6, we have N-Q-cl(N-Q-cl A) ⊆A. Since N-Q-int(N-Q-cl A)⊆ N-Q-cl(N-Q-clA), N-Q-int(N-Q-cl A) ⊆A. Hence A is N-Q-semi closed.

Remark 6.4:

Converse of the above Theorem need not be true.N-Q- pre intB.

Example 6.5: in Example 6..2

Clearly A = {b,d} is N-Q-semi open Clearly A = {b, d} is N-Q-semi closed, but not N-Q-closed.

7.Continuous Functions In N-Q-Topological Spaces

Definition 7.1:

Let $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$ and $\left(V, \sigma_{R_{1,2,3,4}}(X)\right)$ be two N-Q-topological spaces. A function f: U→V is called N-Q-continuous function if $f^{-1}(H)$ is N-Q-open in U for every N-Q- open set H in V.

Example 7.2:

Let U = {p, q , r, s, t}, U/R_1= { {p} , {q , r, s}, {t}}.

LetX_1={p,q}⊆U.The$\tau_{R_1}(X)$={U, ϕ ,{p},{p,q,r,s},{q,r,s}}

LetX_2={p,r}⊆U.The$\tau_{R_2}(X)$={U, ϕ ,{p},{p,q,r,s},{q,r,s}}.

Let X_3 = {q , r}⊆ U. Then $\tau_{R_3}(X)$={U, ϕ ,{q,r,s} }.

Let X_4 = {q , s}⊆ U. Then $\tau_{R_4}(X)$={U, ϕ ,{q,r,s} }.

Then N-Q- O(X)= {U, ϕ , {p},{p,q,r,s},{q,r,s}}

Let V = {1, 2 , 3, 4, 5}, U/R_1= { {1,2,3} , {4}, {5}}.

Let X_1 ={1 , 2}⊆ U. Then $\sigma_{R_1}(X)$={V, ϕ ,{1,2,3}}

Let X_2 = {2 , 3}⊆ U. Then $\sigma_{R_2}(X)$={V, ϕ ,{1,2,3 }}.

LetX_3={1,4}⊆U.The$\sigma_{R_3}(X)$={V, ϕ,{4},{1,2,3,4},{1,2,3}}

LetX_4={3,4}⊆U.The$\sigma_{R_4}(X)$={V, ϕ {4},{1,2,3,4},{1,2,3}}

Then N-Q- O(X)= {U, ϕ ,{4},{1,2,3,4},{1,2,3} }.

f:U→V be a function defined as f(p) = 4, f(q) = 2, f(r)=3,f(s)=1.N-Q-opensetsin U are {p},{p,q,r,s},{q,r,s}}and N-Q-open sets in V are {4},{1,2,3,4},{1,2,3} .Therefore for every N-Q-open set H in V, $f^{-1}(H)$ is N-Q-open set in U. Then f is N-Q-continuous function.

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Definition 7.3:
Let U and V be the two N-Q-topological space. A function f: U→V is called N-Q-continuous at a point a ∈ U if for every N-Q-open set H containing f(a) in V, there exist a N-Q-open set G containing a in U, such that f(G)⊂ H.

Theorem 7.4:
f: U→V is N-Q-continuous if and only if f is N-Q-continuous at each point of U.

Proof:
Let f: U→V be N-Q-continuous. Let a ∈ U, and H be a N-Q-open set in V containing f(a). Since f is N-Q-continuous, $f^{-1}(V)$ is N-Q-open in U containing a. Let $G=f^{-1}(H)$, then f(G) ⊂ H, and f(a) ∈ G. Hence f is continuous at a.

Conversely, suppose f is N-Q-continuous at each point of U. Let H be N-Q-open set in V. If $f^{-1}(H) = \phi$ then it is N-Q-open. So let $f^{-1}(H) \neq \phi$. Take any a ∈ $f^{-1}(H)$, then f(a) ∈ H. Since f is N-Q-continuous at each point there exist a N-Q-open set G_a containing a such that f (G_a)⊂H. Let Let G = (Ga |a ∈ $f^{-1}(H)$).

Claim: G = f^{-1} (H) If x ϵ f^{-1} (H) then x ϵ Gx⊂G. Hence f^{-1} (H)⊂G. On the other hand, suppose yϵG then y ϵ Gx for some x and y ϵ f^{-1} (H). Hence U = f^{-1} (H). Since Gx is N-Q-open, by Theorem 3.1.7 (i) G is N-Q-open and hence G = f^{-1} (H) is N-Q-open for every N-Q-open set H in V. Hence f is N-Q-continuous.

Theorem 7.5: Let $(U, \tau_{R_{1,2,3,4}}(X))$ and $(V, \sigma_{R_{1,2,3,4}}(X))$ be two N-Q-topological spaces. Then f: U→V is N-Q-continuous function if and only if f^{-1} (H) is N-Q- closed in U, whenever H is N-Q- closed in V.

Proof:
Let f: U→V is N-Q-continuous function and H be N-Q-closed in V. Then H^C is N-Q-open inV. By hypothesis $f^{-1}(H^C)$ is N-Q-open in U, i.e., $[f^{-1}(H)]^C$ is
N-Q-open in U. Hence
f^{-1} (H) is N-Q-closed in U whenever H-is N-Q- closed in V. Conversely, suppose $f^{-1}(H)$ is N-Q- closed in U whenever H is N-Q-closed in V. Let U is N-Q-open in V then G^C is N-Q-closed in V. By assumption f^{-1} (G^C) is N-Q-closed in U. i.e., $[f^{-1} (G)]^C$ is N-Q-closed in X. Then
f^{-1} (G) is N-Q-open in U. Hence f is N-Q-continuous.

Theorem 7.6: Let $(U, \tau_{R_{1,2,3,4}}(X))$ and $(V, \sigma_{R_{1,2,3,4}}(X))$ be two N-Q-topological space. Then
f: U→V is N-Q-continuous function if and only if f(N-Q-cl A) ⊂N-Q-cl [f(A)].

Proof:
Suppose f: U→V is N-Q-continuous and N-Q- cl [f(A)] is N-Q-closed in V. Then by Theorem 7.5, f^{-1} (N-Q- cl [f(A)]) is N-Q-closed in U. Consequently, N-Q- cl[f^{-1}(N-Q- cl [f(A)])] = f^{-1}(N-Q- cl [f(A)]). Since f(A) ⊂N-Q- cl [f(A)], A ⊂f^{-1} (N-Q- cl [f(A)]) and N-Q- cl(A) ⊂N-Q-cl (f^{-1} N-Q- cl [f(A)])) = f^{-1} (N-Q- cl [f(A)]) Hence f (N-Q- cl (A)) ⊂N-Q- cl [f(A)].

Conversely, if f(N-Q- cl (A)) ⊂N-Q-cl [f(A)] for all A ⊂ U. Let F be N-Q-closed set in V, so that N-Q-cl(F)=F …….(1)

By hypothesis, f(N-Q- cl (f^{-1} (F)) ⊂N-Q- cl [f(f^{-1} (F))] ⊂N-Q- cl (F), then by (1), N-Q- cl (f^{-1} (F))⊂F. It follows that N-Q- cl (f^{-1} (F)) ⊂f^{-1} (F). But always f^{-1} (F) ⊂N-Q- cl (f^{-1} (F)], so that N-Q- cl (f^{-1} (F)) = f^{-1} (F). Hence f^{-1}(F) is N-Q-closed in U and f is continuous by Theorem 7.5.

Theorem 7.7
Let $(U, \tau_{R_{1,2,3,4}}(X))$, $(V, \sigma_{R_{1,2,3,4}}(X))$ and $(W, \rho_{R_{1,2,3,4}}(X))$ be three N-Q-topological spaces. If f: U→V and g: V→W are N-Q-continuous mappings then g∘f: U→W is also N-Q- continuous.

Proof:
Let G be a N-Q-open set in W. Since by g is N-Q-continuous, $g^{-1}(G)$ is N-Q-open set in V. Now, $(g∘f)^{-1}G = (f^{-1}∘g^{-1})G = f^{-1} (g^{-1}(G))$. Take $g^{-1}(G)$ = H which is N-Q-open in V, then $f^{-1}(H)$ is N-Q-open in U, since by f is N-Q-continuous. Hence g∘f: U→W is N-Q-continuous function.

8 N-Q-Pre Continuous And N-Q-Semi Continuous Functions

Definition 8.1:
Let $(U, \tau_{R_{1,2,3,4}}(X))$ and $(V, \sigma_{R_{1,2,3,4}}(X))$ be two N-Q-topological spaces, then f: U→V is N-Q-pre continuous if $f^{-1}(V)$ is N-Q- pre closed in U whenever V is N-Q-closed.

Example 8.2
U = {a, b,c}, with U/R= {{a},{b, c}} and

U/R_1={{a},{b, c}} X_1={a, b}⊆ U$\tau_{R_1}(X)$={U,ϕ,{a},{b, c}}

U/R_2= {{b},{a, c}}X_2= {b, c}⊆U $\tau_{R_2}(X)$={U,ϕ,{b},{a,c}}

U/R_3= {{c},{a,b}}X_3={b, c}⊆ U$\tau_{R_3}(X)$={U, ϕ, {c}{a, b}}

U/R_4= {{c},{a, b}} X_4= {a,b}⊆ U $\tau_{R_4}(X)$= {U, ϕ, {a,b}}

Then N-Q- C(X)= {U, ϕ, {a},{b},{c},{a,c}, {a,b},{b,c}}
U = {1,2,3}, with U/R= {{1},{2, 3}} and

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$U/R_1=\{\{1\},\{2,3\}\}$ $X_1=\{1,2\}\subseteq U$ $\tau_{R_1}(X)= U, \phi,\{1\},\{2,3\}\}$

$U/R_2= \{\{1,3,\},\{2\}\}$ $X_2= \{1,3\}\subseteq$ $\tau_{R_2}(X)= \{U, \phi, \{1,3\}\}$

$U/R_3= \{\{1\},\{2,3\}\}$ $X_3= \{2,3\}\subseteq U$ $\tau_{R_3}(X)= \{U, \phi, \{2,3\}\}$

$U/R_4= \{\{3\},\{1,2\}\}$ $X_4= \{3\}\subseteq U$ $\tau_{R_4}(X)= \{U, \phi, \{3\}\}$

Then N-Q- $C(X)= \{U, \phi, \{1\},\{2\},\{1,2\}, \{2,3\}\}$
let f: U→V be a function defined as f(a) = 1, f(b) = 2, f(c) = 3. Here N-Q-closed sets in Y are {2} and {1, 2}. Then the inverse images of these sets are {b}, {a, b} and they are N-Q- pre closed in X. Hence f is N-Q-pre continuous.

Theorem 8.3: Every N-Q-continuous function is N-Q-pre continuous

Proof:

Let f: U→V be N-Q-continuous. i.e., $f^{-1}(H)$ is N-Q-closed in U, whenever H is N-Q-closed in V. By Theorem 3.3.3, every N-Q-closed set is N-Q-pre closed, and hence $f^{-1}(V)$ is N-Q- pre closed in U whenever H is closed in V. Hence f: U→V be N-Q-pre continuous

Definition 8.6:

Let $\left(U, \tau_{R_{1,2,3,4}}(X)\right)$ and $\left(V, \sigma_{R_{1,2,3,4}}(X)\right)$ be two N-Q-topological space, then f: U→V is N-Q-semi continuous if $f^{-1}(H)$ is N-Q- semi closed in U whenever H is closed in V.

Example 8.7:

In Example 8.2 N-Q-closed sets in V are {2} and {1,2}. Then the inverse images of these sets are {b}, {a, b} and they are N-Q- semi closed in U. Hence f is N-Q-semi continuous.

Theorem 8.8:

Every N-Q-continuous function is N-Q-semi continuous.

Proof:

Let f: U→V be N-Q-continuous. i.e.,$f^{-1}(H)$ is N-Q- closed in U, whenever H is N-Q-closed in V. By Theorem 4.3 (ii), every N-Q-closed set is N-Q-semi closed. This implies that $f^{-1}(H)$ is N-Q- semi closed in U whenever H is closed in Y. Hence f: U→V be N-Q-semi continuous.

9. Conclusions

In this paper we introduced new type topology is called Nano Quad topology. and also we introduce the concepts of N-Q-pre open.and N-Q-semi open and some of their properties are studied .then we proved Every N-Q-open set is N-Q-pre open and Every N-Q-open set is N-Q-semi open.N-Q continuity disscssed detaild.. Finally, we hope that this paper is just a beginning of new classes of functions, it will be necessary to carry out more theoretical research to investigate the relations between

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Reference

[1] Abu-Donia H.M., Generalized separation axioms in bitopological space, The Arabian JI for Science and Engineering, 30 (2005), 117-129.

[2] Abu-Donia H.M., New types of generalized closed sets in bitopological space, Journal of the Egyptian Mathematical Society, 21(2013),318-23

[3] Abu-Donia H.M., Generalized ψ*-closed sets in bitopological space, Journal of the Egyptian Mathematical Society, 23 (2015), 527-534.

[4] Allam A.A., New types of continuity and openness in fuzzifying bitopological space, Journal of the Egyptian Mathematical Society, 24 (2016), 286-294.

[5] Bhattacharya B.,Paul.A., A New Approach of γ-Open Sets in bitopological spaces, Gen Math. Notes, 20 (2014), 95-110.

[6] James R.Munkres., Topology, Pearson Education, Inc., Prentice Hall (2013).

[7] Kelly J. C., Bitopological Spaces, Proc. London Math. Soc., 3 (1963), 17-89.

[8] Kovar M., On 3-Topological version of Theta regularity, Internat. J. Math. Sci., 23 (6) (2000), 393-398.

[9] Palaniammal S., Murugalingam S., A Study of Tri Topological Spaces, Shodhganga, a reservoir of Indian thesis., (2014).

[10] N. Levine, **1963**, *"Semi-open sets andsemi-continuity in topological spaces"*,*Amer. Math. Mothly*,Vol. 70, 36-41.

[11] J.Stella irene mary j.,M. Hemalatha On tri star topological spaces induced by bitopological spaces",Internat. J. Math.Archive Vol.7(9), 2016,p 147-153

[12] S.Chandrasekar, T.Rajesh kannan , M.Suresh Nano(1, 2)*Locally Closed Sets in Nano Bitopological spaces communicated

[13] S.Chandrasekar, T.Rajesh kannan , M.Suresh Contra-Nanosg-Continuity inNanoTopological Spaces 2017Vol.(2)110-116International Journal on Research Innovations in Engineering Science and Technology

[14] M.Lellis Thivagar, and Carmel Richard, On Nano Forms of Weakly Open sets, International JournalMathematicsandStatistics Invention,(2013), 31-37

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[15] K. Bhuvaneswari1, H. Rasya Banu2.On Nano (1,2)* Generalized - Closed Sets in Nano Bitopological Space Int.J. Science and Research Volume 5, Apr 2016,P 1292-1295

[16] S.Chandrasekar,V.Banupriya,J.Sureshkumar Nano b - open sets in nano tri star topological spaces Accepted in International Journal of Scientific Engineering and Applied Science

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