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Determination of Fractional Chromatic Number on Two Different Graphs of Amalgamation Operation

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Abstract - The main problem in vertex coloring is how to color each vertex on a graph so that no two adjacent vertices have the same color. Fractional coloring is a double coloring at vertices of different colors where the adjacent vertices have different colors. In operations on the graph, one of them is known as amalgamation operation. Amalgamation in the graph is divided into two, namely vertex amalgamation and side amalgamation. Coloring vertex can be applied to the graph which is the result of the operation of some special graphs. Amalgamation operation is to combine the vertices or sides of each graph to form a new graph. In this case, the resulting amalgamation graph will produce the same fractional chromatic number with one of the fractional chromatic figures of the graph prior to be amalgamated.

Keywords *- Amalgamation, Fractional Chromatic Number, Graph Coloring.*

1. Introduction

The development of graph theory has given much input to the new science; one of which is graph coloring. Its applications are used in various fields such as the theory of coding index, *X-ray* crystallography, Astronomy, and a communication network system.

In graph coloring, there are three kinds of coloring: vertex coloring, edge coloring, and face coloring. The main problem in vertex coloring is how to color all the vertices on the graph so that no two adjacent vertices have the same color. Fractional coloring is a double coloring on vertices with different colors and adjacent vertices having different colors.

Fractional chromatic number is obtained after we give a double color to each vertex in the graph so that each vertex is colored differently if the vertex is adjacent and the number of minimum colors used is called fractional chromatic number. The previous researchers only explain how to obtain fractional chromatic number from a single graph type; it has not yet explained the fractional chromatic number of the operation result graph from two graphs constructed by different graphs.

In this journal, the researchers propose for the study of chromatic fractional number of grafts resulting from amalgamation operation of two different graphs in which it will produce a new graph so that the chromatic number of the graph will be influenced by one of the fractional chromatic numbers of one of the graphs prior to be amalgamated.

To understand more about the coloring of graphs and the operation of amalgamation, there are several definitions and theorems related to the discussion as follows:

Definition 1 *Vertex coloring is to give color to the vertices on the graph so that every two adjacent vertices have different colors.*

Definition 2 *Suppose* $I: V \rightarrow [N]^k$ *a mapping. Mapping* I *is called coloring-k vertex graph* $G(V,E)$*, if for each* $u, v \in V(G)$*, with* $uv \in E(G)$*,* $I(u) \neq I(v)$ applies. *(Karthikeyan Shanmugam. 2013).*

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Definition 3 *a non-directional graph G = (V,E) is given, the free set is a subset of vertices $U \subseteq V$, so that there are no two vertices in the adjacent U. (J. A. Bondy, and U. S R. Murty. 1982)*

Definition 4 *Fractional chromatic numbers in simple graphs $G(V, E)$ defined with $\chi_f(G)$ is given by* $min \sum_{I \in \mathcal{I}} x_I$

$$\sum_{I: v \in I} x_I \geq 1, v \in V, x: [N]^k \to \mathbb{R}^+$$

$$x_I \in \mathbb{R}^+, \forall I \in \mathcal{I}$$

$$x(u_i) = \left| \frac{A_i}{\bigcup_{j=1}^{i-1} A_j} \right|, \forall i, j \in \mathbb{N}$$

In which I is the set of all independent sets in G, $\mathbb{R}^+$ is a positive real number.
. (Karthikeyan Shanmugam.2013)

Theorem 1 *If there is a coloring-k in graph G, then $\chi_f(G) \leq k$. (Pirnazar and Ullman. 2002).*

Definition 5 *Suppose G_i is a graph with a fixed vertex v_{0i} for $i = \{1,2, \ldots, k\}$. Vertex amalgamation (G_i, v_{0i}) is a graph formed by taking all the graphs G_i Where $v_{0i} = v_{0j}, \forall i, j \in \{1,2, \ldots, k\}$. (Diestel. 2005)*

Example 1

Amalgamation of path and circle graphs

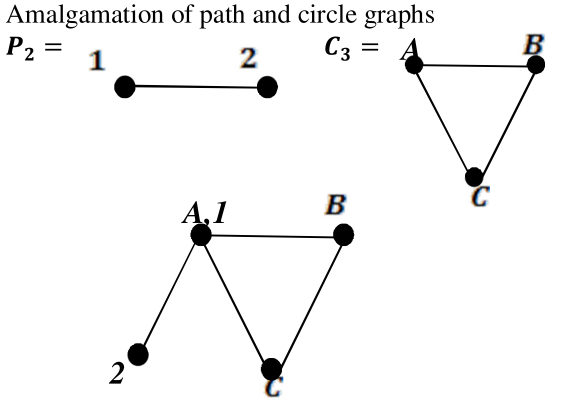

Figure 1 Example of Vertex Amalgamation

Definition 6 *Suppose G_i is a graph with side $e_{i1}, e_{i2}, \ldots, e_{ij}$ in $E(G_i)$ and G_r with side$e_{r1}, e_{r2}, \ldots, e_{rk}$ in $E(G_r)$. Side amalgamation $G_i$to G_r is taking all G_i and G_r with $e_{is} = e_{rs}$ for $s \leq j, k$. Side amalgamation G_i towards G_r is denoted $(G_i, e_{is}: G_r, e_{rs})$. (Diestel. 2005)*

Example 2

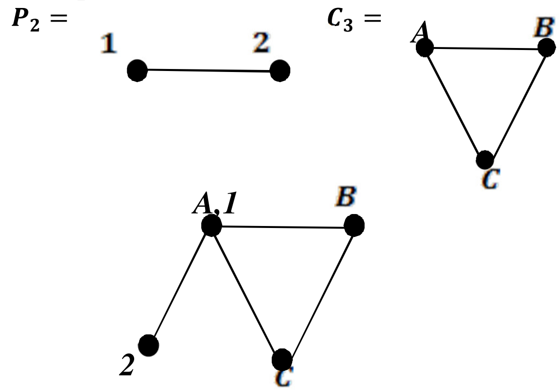

Figure 2 Example of Side Amalgamation

Side amalgamation P_3 towards C_3 is denoted $P_3, e_{11}: C_3, e_{21})$ Where $e_{11} = e_{21}$ or $e_{11} = 2A = 1B = e_{21}$.

2. Result and Discussion

The previous studies search chromatic fractional graph number using the same method which is literature study, where the difference is on the construction of the graph. The steps used in this research are:

1. Collecting literature materials and library studies related to the graph.
2. Drawing unconnected graph without loop for and with n as vertex and m is line.
3. Grouping unrelated graphs for the same maximum line.
4. Counting the number of unconnected graphs for each line.
5. Seeing the pattern of the number of graphs that are formed.
6. Coloring the vertex of the graph (more than one color in 1 vertex)
7. Making a conclusion.

It is because of the more graphic construction, the more fractional chromatic numbers are found. Furthermore, the fractional chromatic number of each graph is less than or equal to the chromatic number. It is also known well that the fractional chromatic number in each graph is in integers but there is also in fractional number. Fractional coloring is a topic in a new branch of graph theory known as the fractional graph theory. It is a generalization of ordinary graph coloring.

In the fractional graph coloring, each vertex in the graph is given several colors, and the vertices that are intertwined or connected by the sides must be colored differently. However, in fractional coloring, one set is given the same color at each vertex of the graph. Fractional graph coloring can be seen as a linear programming relaxation of graph coloring. Indeed, the problem of fractional coloring is much better used for a linear programming approach than ordinary coloring problems.

In accordance with the needs of the graph, graph is frequently constructed so that to form a graph that can be applied in the world of technology, it is often used in the graph operation to construct. Moreover, the types of graph operations are Joint (G + H), Crown Product (G ⨀ H), Tensor Product (G ⊗ H), Cartesian product (GH), Composition (G [F]), Shackle, and Amalgamation.

The first graph found for fractional chromatic number is Roberts' cocktail party graph [15], Roberts explains that coloring at the vertex where each vertex is colored more than one color will produce fractional chromatic number but the drawback is not to explain deeply about the coloring problem. In 1979, Bollobás *et all*. [3], also found chromatic fractional web graphics, helmets, bipartite and tripartite figures by explaining in more detail that fractional chromatic numbers are not only integers but also in fractions. Afterwards, the chromatic number of the pan graph is found where Saaty *et all*. [16] applied four colors in a single vertex.

With many researchers in the field of graphs, many are interested in graph coloring. In 1990, Skiena [18] found fractional chromatic numbers in cycle graphs, stars and, wheels while explaining comparisons between chromatic numbers and fractional chromatic numbers, where Skiena states that chromatic numbers are always greater or equal to fractional chromatic numbers. Furthermore, Larsen and Ullman [12] develops the construction of Mycielski's graph and searches for fractional chromatic numbers, until now Ullman still wrestles this field where he is now preparing a book called "Fractional Graph Theory" which contains the whole discussion of fractional coloring.

Godsil and Royle [9] also examine fractional chromatic numbers from trajectory graphs by taking the basis of some earlier researchers. As time goes by, Pilnazar and Ullman [14] again finds fractional chromatic numbers on the planar graph by constructing several graphs such as star graphs and complete graphs. Golin [10] also, in 2004, developed fractional coloring by applying it to an anti prism graph and tree graph then comparing the results.

With the development of many new graph constructions, the researchers in the field of graph coloring are also increasingly active in searching fractional chromatic numbers of the graphs. Ghost et al. [8] in 2006 examined the chromatic number of the barbell graph, then followed by fractional chromatic of complete graph by applying adjacency elements. In the results of this study, Bryant [4] explains that to make it easier to find fractional chromatic numbers, it should make a set of vertices in which the adjacent vertex must have a different set. Subsequently, in 2008 [2] there were further fractional chromatic numbers of the solar graph and the sun graph, followed in 2011, Ullman et al. [17] re-examined fractional chromatic numbers of kneser, blank and prism graphs.

With the development of the world of technology, so many researchers begin to glance at some form of graph that can be applied into modern devices. One of which is fractional chromatic number, where some researchers have not constructed a new graph but looking for where research results on graph coloring can be applied. In 2013, Graph K_Δ- free [7] which is the construction of the shift graph used in the index of coding where it is reviewed is fractional chromatic number. It is explained that to make the most optimal encoding index is by using fractional coloring. Then, Zdenek Dvorak et al. [20] also found fractional chromatic numbers in the Graph of Sub cubic triangle-free where applied to computational programming. In 2014, many researchers began to be interested in constructing several graphs into a new graph in accordance with the desired shape in order to be applied in the index code; one of which is Kneser Hyper Graph [13] which is also the construction of the kneser graph.

To date, many researchers have published journals related to fractional coloring which are applimed in the field of computers; one of which is a journal written by Fatemeh Arbabjolfaei and Young-Han Kim [19] by 2015, they explain that the problem of coded region-specific code indexes from the smallest problem in the event of a non-existent, single-line or complex interaction can be solved by using fractional coloring where small fractional chromatic numbers are helpful in index coding.

The fractional chromatic number of cycle graph examined by Skiena (1990) is proven as follows

a) Suppose cycle graph C_3 is given with $V(C_3) = \{u_1, u_2, u_3\}$ and $E(C_3) = \{u_1u_2, u_2u_3, u_3u_1\}$, the minimum independent set of C_3 is

$I_1 = \{u_1\}$

$I_2 = \{u_2\}$

$I_3 = \{u_3\}$

Then, the set of all independent sets is $\mathcal{I} = \{I_1, I_2, I_3\}$.

To $I: V(C_3) \rightarrow [N]^2$

$U_1 \rightarrow \{1,2\} = A_1$

$u_2 \rightarrow \{3,4\} = A_2$

$u_3 \rightarrow \{5,6\} = A_3$

$x: [N]^2 \rightarrow \mathbb{R}^+$

$$x(u_1) = \frac{|A_1|}{2} = \frac{2}{2}$$

$$x(u_2) = \frac{|A_2/A_1|}{2} = \frac{2}{2}$$

$$x(u_3) = \frac{|A_3/(A_1 \cup A_2)|}{2} = \frac{2}{2}$$

Then, the fractional number is

$$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_2} x_I = \frac{2}{2} + \frac{2}{2} + \frac{2}{2} = \frac{6}{2} = 3$$

To: $I: V(C_3) \rightarrow [N]^3$

$u_1 \rightarrow \{1,2,3\} = A_1$

$u_2 \rightarrow \{4,5,6\} = A_2$

$u_3 \rightarrow \{7,8,9\} = A_3$

$x: [N]^3 \rightarrow \mathbb{R}^+$

$$x(u_1) = \frac{|A_1|}{3} = \frac{3}{3}$$

$$x(u_2) = \frac{|A_2/A_1|}{3} = \frac{3}{3}$$

$$x(u_3) = \frac{|A_3/(A_1 \cup A_2)|}{3} = \frac{3}{3}$$

Then, the fractional number is

$$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_3} x_I = \frac{3}{3} + \frac{3}{3} + \frac{3}{3} = \frac{9}{3} = 3$$

To: $I: V(C_3) \rightarrow [N]^4$

$u_1 \rightarrow \{1,2,3,4\} = A_1$

$u_2 \rightarrow \{5,6,7,8\} = A_2$

$u_3 \rightarrow \{9,10,11,12\} = A_3$

$x: [N]^4 \rightarrow \mathbb{R}^+$

$$x(u_1) = \frac{|A_1|}{4} = \frac{4}{4}$$

$$x(u_2) = \frac{|A_2/A_1|}{4} = \frac{4}{4}$$

$$x(u_3) = \frac{|A_3/(A_1 \cup A_2)|}{4} = \frac{4}{4}$$

Then, the fractional number is

$$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_4} x_I = \frac{4}{4} + \frac{4}{4} + \frac{4}{4} = \frac{12}{4}$$

To: $I: V(C_3) \rightarrow [N]^5$

$u_1 \rightarrow \{1,2,3,4,5\} = A_1$

$u_2 \rightarrow \{6,7,8,9,10\} = A_2$

$u_3 \rightarrow \{11,12,13,14,15\} = A_3$

$x: [N]^5 \rightarrow \mathbb{R}^+$

$$x(u_1) = \frac{|A_1|}{5} = \frac{5}{5}$$

$$x(u_2) = \frac{|A_2/A_1|}{5} = \frac{5}{5}$$

$$x(u_3) = \frac{|A_3/(A_1 \cup A_2)|}{4} = \frac{5}{5}$$

Then, the fractional number is

$$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_5} x_I = \frac{5}{5} + \frac{5}{5} + \frac{5}{5} = \frac{15}{5} = 3$$

etc.

Therefore, the fractional number is $\{3,3,3,3,\ldots\}$, and its fractional chromatic number is $min \sum_{I:u_i \in I, \forall I \in \mathcal{I}_k} x_I = min\{3,3,3,3,\ldots\} = 3$

to be more clearly, please take a note the following figure

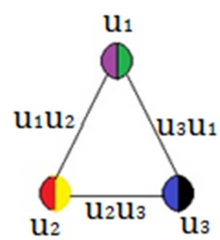

Figure 3.1 Fractional Chromatic Numbers on the Cycle Graph C_3

b) Suppose cycle graph C_4 is given with $V(C_4) = \{u_1, u_2, u_3, u_4\}$ and $E(C_4) = \{u_1u_2, u_2u_3, u_3u_4, u_4u_1\}$, the minimum independent set of C_4 is

$I_1 = \{u_1, u_3\}$

$I_2 = \{u_2, u_4\}$

Then, the set of all independent sets $\mathcal{I} = \{I_1, I_2\}$.

To: $I: V(C_4) \rightarrow [N]^2$

$u_1 \rightarrow \{1,2\} = A_1$

$u_2 \rightarrow \{3,4\} = A_2$

$u_3 \rightarrow \{1,2\} = A_3$

$u_4 \rightarrow \{3,4\} = A_4$

$x: [N]^2 \rightarrow \mathbb{R}^+$

$$x(u_1) = \frac{|A_1|}{2} = \frac{2}{2}$$

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$$x(\mathrm{u}_2) = \frac{|A_2/A_1|}{2} = \frac{2}{2}$$
$$x(\mathrm{u}_3) = \frac{|A_3/(A_1 \cup A_2)|}{2} = \frac{0}{2}$$
$$x(\mathrm{u}_4) = \frac{|A_4/\cup_{i=1}^{3} A_i|}{2} = \frac{0}{2}$$

Then, the fractional number is

$$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_2} x_I = \frac{2}{2} + \frac{2}{2} + \frac{0}{2} + \frac{0}{2} = \frac{4}{2} = 2$$

To: $I: V(C_3) \rightarrow [N]^3$
$\mathrm{u}_1 \rightarrow \{1,2,3\} = A_1$
$\mathrm{u}_2 \rightarrow \{4,5,6\} = A_2$
$\mathrm{u}_3 \rightarrow \{1,2,3\} = A_3$
$x: [N]^3 \rightarrow \mathbb{R}^+$

$$x(\mathrm{u}_1) = \frac{|A_1|}{3} = \frac{3}{3}$$
$$x(\mathrm{u}_2) = \frac{|A_2/A_1|}{3} = \frac{3}{3}$$
$$x(\mathrm{u}_3) = \frac{|A_3/(A_1 \cup A_2)|}{3} = \frac{0}{3}$$
$$x(\mathrm{u}_4) = \frac{|A_4/\cup_{i=1}^{3} A_i|}{3} = \frac{0}{3}$$

$\mathrm{u}_4 \rightarrow \{4,5,6\} = A_4$

Then, the fractional number is
$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_3} x_I = \frac{3}{3} + \frac{3}{3} + \frac{0}{3} + \frac{0}{3} = \frac{6}{3} = 2$

To: $I: V(C_3) \rightarrow [N]^4$
$\mathrm{u}_1 \rightarrow \{1,2,3,4\} = A_1$
$\mathrm{u}_2 \rightarrow \{5,6,7,8\} = A_2$
$\mathrm{u}_3 \rightarrow \{1,2,3,4\} = A_3$
$\mathrm{u}_4 \rightarrow \{5,6,7,8\} = A_4$
$x: [N]^4 \rightarrow \mathbb{R}^+$

$$x(\mathrm{u}_1) = \frac{|A_1|}{4} = \frac{4}{4}$$
$$x(\mathrm{u}_2) = \frac{|A_2/A_1|}{4} = \frac{4}{4}$$
$$x(\mathrm{u}_3) = \frac{|A_3/(A_1 \cup A_2)|}{4} = \frac{0}{4}$$
$$x(\mathrm{u}_4) = \frac{|A_4/\cup_{i=1}^{3} A_i|}{4} = \frac{0}{4}$$

Then, the fractional number is
$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_4} x_I = \frac{4}{4} + \frac{4}{4} + \frac{0}{4} + \frac{0}{4} = \frac{8}{4} = 2$

To: $I: V(C_3) \rightarrow [N]^5$
$\mathrm{u}_1 \rightarrow \{1,2,3,4,5\} = A_1$
$\mathrm{u}_2 \rightarrow \{6,7,8,9,10\} = A_2$
$\mathrm{u}_3 \rightarrow \{1,2,3,4,5\} = A_3$
$\mathrm{u}_4 \rightarrow \{,7,8,9,10\} = A_4$
$x: [N]^5 \rightarrow \mathbb{R}^+$

$$x(\mathrm{u}_1) = \frac{|A_1|}{5} = \frac{5}{5}$$
$$x(\mathrm{u}_2) = \frac{|A_2/A_1|}{5} = \frac{5}{5}$$
$$x(\mathrm{u}_3) = \frac{|A_3/(A_1 \cup A_2)|}{5} = \frac{0}{5}$$
$$x(\mathrm{u}_4) = \frac{|A_4/\cup_{i=1}^{3} A_i|}{5} = \frac{0}{5}$$

Then, the fractional number is
$\sum_{I:u_i \in I, \forall I \in \mathcal{I}_5} x_I = \frac{5}{5} + \frac{5}{5} + \frac{0}{5} + \frac{0}{5} = \frac{10}{5} = 2$, dst.

Hence, the fractional number is $\{2,2,2,2, \ldots\}$, and its fractional chromatic number is $min \sum_{I:u_i \in I, \forall I \in \mathcal{I}_k} x_I = min\{2,2,2,2, \ldots\} = 2$

to be more clearly, please take a note the following figure

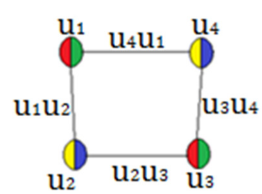

Figure 3.2 Fractional Chromatic Numbers on the Cycle Graph C_4

The relevant researches related to the fractional chromatic number in the graph are as follows:

Table 1 Previous relevant studies

No.	Research Title (Year)	Graph Name	Symbol	Fractional chromatic number
1	[15] On the Boxicity and Cubicity of a Graph (1969)	Cocktail Party Graph	$CP_n, n \geq 1$	n
2	[3] Set Colorings of Graphs (1979)	Helmet Graph	$\bar{H}_{2n+1}, n \in \mathbb{N}$	$3 + \frac{1}{n}$
			$\bar{H}_{2(n+1)}, n \in \mathbb{N}$	3
3	[3] Set Colorings of Graphs (1979)	Web Graph	$\bar{W}_{2n+1}, n \in \mathbb{N}$	$2 + \frac{1}{2n}$
			$\bar{W}_{2(n+1)}, n \in \mathbb{N}$	2
4	[3] Set Colorings of Graphs (1979)	Bipartite Graph	$B_n, n \geq 2$	2
5	[3] Set Colorings of Graphs (1979)	Tripartite Graph	$\bar{T}_n, n \geq 3$	3
6.	[14] The Four-Color Problem: Assaults and Conquest (1986)	Pan Graph	$\bar{P}_{2n+1}, n \in \mathbb{N}$	2
			$\bar{P}_{2(n+1)}, n \in \mathbb{N}$	$2 + \frac{1}{n}$
7.	[18] Cycles, Stars, and Wheels. "In Computational Discrete Mathematics: Combinatorics and Graph Theory in Mathematica "(1990)	Cycle Graph	$C_{2n+1}, n \in \mathbb{N}$	$2 + \frac{1}{n}$
			$C_{2(n+1)}, n \in \mathbb{N}$	2
8	[18] Cycles, Stars, and Wheels. "In Computational Discrete Mathematics: Combinatorics and Graph Theory in Mathematics "(1990)	Wheel Graph	$Y_{2n+1}, n \in 2,3 \ldots$	3
			$Y_{2(n+1)}, n \in \mathbb{N}$	$3 + \frac{1}{n}$
9	[18] Cycles, Stars, and Wheels. "In Computational Discrete Mathematics: Combinatorics and Graph Theory in Mathematica "(1990)	Star Graph	$S_n, n \geq 4$	2
10	[12] The Fractional Chromatic Number of Mycielski's Graphs (1995)	Mycielski Graph	M_n	$a_2 = 2, dan\ a_n = a_{n-1} + \frac{1}{a_{n-1}}$
11	[9] Fractional Chromatic Number (2001)	Track Graph	P_n	2

12	[14] Girth and Fractional Chromatic Number of Planar Graphs (2002)	Planar Graph	$\bar{Pl}_n$	$3 - \frac{1}{n + \frac{1}{3}}$
13	[10] Unhooking Circular Graphs: a Combinatorial Method for Counting Spanning Trees and Other Parameters (2004)	Anti Prism Graph	$\bar{Y}_n, n \geq 3$	3, 4, 10/3, 3, 7/2, 16/5, 3, 10/3, ...
14	[9] Unhooking Circular Graphs: a Combinatorial Method for Counting Spanning Trees and Other Parameters (2004)	Tree Graph	$T_n, n \geq 3$	2
15	[8] Minimizing Effective Resistance of a Graph (2006)	Barbell Graph	$\bar{B}_n, n \geq 3$	n
16	[4] Cycle Decompositions of Complete Graphs (2007)	Complete Graph	$K_n, n \geq 3$	n
17	[2] N-Sun Decomposition of Complete, Complete Bipartite and Some Harary Graphs (2008)	The Sun Graph	$\bar{S}_n, n \geq 3$	n
18	[2] N-Sun Decomposition of Complete, Complete Bipartite and Some Harary Graphs (2008)	Sunlet Graph	$\bar{Sl}_{2n+1}, n \in \mathbb{N}$	2
			$\bar{Sl}_{2(n+1)}, n \in \mathbb{N}$	$2 + \frac{1}{n}$
19	[17] Fractional Graph Theory A Rational Approach to the Theory of Graphs (2011)	Kneser Graph	$K(n,k), k < n-1$	$\frac{n}{k}$
20	[17] Fractional Graph Theory A Rational Approach to the Theory of Graphs (2011)	Blank Graph	$\bar{K}_n$	1
21	[17] Fractional Graph Theory A Rational Approach to the Theory of Graphs (2011)	Prism Graph	$Y_{2n+1}, n \in \mathbb{N}$	2
			$Y_{2(n+1)}, n \in \mathbb{N}$	$2 + \frac{1}{n}$
22	[7] Bounding the fractional chromatic number of Kâˆ† -free graphs (2013)	K_Δ-free Graph	$K_\Delta, \Delta\{6,7,8\}$	$\Delta - \frac{1}{2}$
23	[20] Subcubic triangle-free graphs have fractional chromatic number at most 14/5 (2013)	Subcubic triangle-free graph	$\bar{S}_n$	$\frac{14}{5}$

3. Closing

From the description or explanation above, the researchers can conclude that fractional chromatic numbers are very useful in the world of technology, especially in the field of index code. The fractional chromatic number of some graphs when combined using an amalgamation operation will result in the same as one chromatic fractional graph before the operation.

Realizing that the researchers are still far from perfection, the researchers will be more focused and detailed in the future regarding the review of fractional chromatic number above with more sources which absolutely can be accounted for and it is suggested to other researchers who are interested in graph, especially coloring, to study fractional chromatic number resulting from other operations on the graph in order to obtain new graph constructions.

References

[1] Aigner, M. and Ziegler, G. M. "The Chromatic Number of Kneser Graphs.". New York: Springer-Verlag, 2000.

[2] Anitha, R. and Lekshmi, R. S. "N-Sun Decomposition of Complete, Complete Bipartite and Some Harary Graphs." *Int. J. Math.*, 2008.

[3] Bollobás, B. and Thomassen, A. "Set Colorourings of Graphs." *Disc. Math*, 1979.

[4] Bryant, D. E. "Cycle Decompositions of Complete Graphs." In *Surveys in Combinatorics*. Cambridge, England: Cambridge University Press, 2007.

[5] Chartrand, G. *Introductory Graph Theory.* New York: Dover, 1985.

[6] Diestel, R. *Graph Theory*. Graduate Texts in Mathematics. 2005

[7] Edwards Katherine. Bounding the fractional chromatic number of K_{Δ}-free graphs. Department of Computer Science Princeton University, Princeton, NJ. 2013

[8] Ghosh, A.; Boyd, S.; and Saberi, A. "Minimizing Effective Resistance of a Graph." *Proc. 17th Internat. Sympos. Math. Th. Network and Systems, Kyoto, Japan, 2006.*

[9] Godsil, C. and Royle, G. "Fractional Chromatic Number." in *Algebraic Graph Theory*. New York: Springer-Verlag, 2001.

[10] Golin, M. J. and Leung, Y. C. "Unhooking Circulant Graphs: a Combinatorial Method for Counting Spanning Trees and Other Parameters." In *Graph-Theoretic Concepts in Computer Science. Revised Papers from the 30th International Workshop WG, 2004*

[11] Karthikeyan Shanmugam. 2013. *Local Graph Coloring and Index Coding*. Department of Mathematics and Computer Science the Open University of Israel, Israel

[12] Larsen, M.; Propp, J.; and Ullman, D. "The Fractional Chromatic Number of Mycielski's Graphs." *J. Graph,* 1995

[13] Pardo G. Araujo; Diaz P. J. Carlos. "The (p,q)-extremal problem and the fractional chromatic number of Kneser hypergraphs." Instituto de Matem´aticas Universidad Nacional Aut´onoma de M´exico Ciudad Universitaria, 2014

[14] Pirnazar, A. and Ullman, D. H. "Girth and Fractional Chromatic Number of Planar Graphs." *J. Graph*, 2002.

[15] Roberts, F. S. "On the Boxicity and Cubicity of a Graph." In *Recent Progress in Combinatorics.* New York: Academic Press, 1969.

[16] Saaty, T. L. and Kainen, P. C. *The Four-Color Problem: Assaults and Conquest.* New York: Dover, p. 148, 1986.

[17] Scheinerman, E. R. and Ullman, D. H. *Fractional Graph Theory A Rational Approach to the Theory of Graphs.* New York: Dover, 2011.

[18] Skiena, S. "Cycles, Stars, and Wheels." in *Implementing Discrete Mathematics: Combinatorics and Graph Theory with Mathematica.* Reading, MA: Addison-Wesley, 1990.

[19] Young-Han Kim, and Fatemeh Arbabjolfaei. Structural Properties of Index Coding Capacity Using Fractional Graph Theory. University of California, San Diego. 2015

[20] Zdenek Dvorak. Jean Sebastien and Sereni Jan Volec. Subcubic triangle-free graphs have fractional chromic number at most 14/5. 2013